(Re)Balancing Act: The Interplay of Private and Public Assets in Dialing the Asset Allocation

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Abstract

The growing trend of sovereign wealth and pension funds to allocate more towards private investments has made the management of asset allocation more complex. Traditional rebalancing methods, such as fixed weights rebalancing, encounter problems when applied to private assets, as their illiquidity and lags in appraisal valuations pose challenges. During financial crises, the delayed and smoothed valuations of private assets lead them to be overweight in portfolios, as public assets decline in values. Rebalancing the underweight public assets can increase leverage usage and, more importantly, deteriorate the fund's liquidity position. To address these challenges, this article proposes a holistic rebalancing strategy: rebalance a portfolio to the desired factor allocation by complementing the factor exposures of existing private assets with an allocation to public assets that overall delivers the required factor allocation. This approach safeguards the liquidity position of a fund during market downturns by maintaining a more stable risk and leverage profile. It presents a more dynamic and risk-aware approach for rebalancing portfolios with private assets.

Key Takeaways

- When portfolios include illiquid private assets, standard rebalancing strategies can unintentionally introduce leverage due to the illiquid nature and potential stale valuation of private holdings and lead to a deterioration of the fund's liquidity position. During market downturns, private assets can become significantly overweight due to stale private asset valuations and the depreciation of public assets.

- We develop a methodology –labelled Factor rebalancing strategy– designed to help the portfolio achieve more stable profiles in terms of leverage, risk, and liquidity. We achieve this by considering public assets as complements to the illiquid private assets and making adjustments to the allocations of public assets to maintain the desired factor allocation for the overall portfolio.

- In our historical analysis, the Factor rebalancing strategy successfully navigates the 2008 Global Financial Crisis and the Crash of 2022 by maintaining a consistent level of portfolio leverage and risk. Importantly, this strategy also effectively manages the fund's liquidity position during both crises.

Keywords: Portfolio Construction, Portfolio Theory, Asset Allocation

JEL Classification: G11

Over the past decade, private investments have attracted substantial capital from pension funds and sovereign wealth funds worldwide. According to the 2022 annual report by Global SWF, 18% of the $ 32 trillion in assets collectively held by global pension and sovereign wealth funds were invested in private equity, real estate, and infrastructure.[1] In North America, pension funds now allocate approximately 30% of their portfolios to private investments, a substantial increase from around 10% before the 2008 Global Financial Crisis.[2] In Canada, the so-called 'Maple 8' –a term referring to the eight largest pension funds– allocate between 40% and 50% of their investments to private assets.[3]

As asset owners globally increase their investments in private assets, efficiently managing portfolio allocation becomes ever more important and especially so for funds with strict risk and liquidity controls. Within the group of Canadian 'Maple 8' pension funds, a key focus revolves around portfolio risk and liquidity management during a severe market downturn. This concern arises because the portfolio weights of private assets may rise significantly due to the 'denominator effect'.[4] With a corresponding underweight in public assets, these investors face a difficult decision on how best to reposition the allocation to public assets. This decision is complex because rebalancing (e.g., purchase stocks) during a market downturn could require additional leverage and drain the liquidity reserve. Adding leverage and draining the liquidity reserve for rebalancing during a severe market downturn can be the one step that institutional investors are not willing to take, as other sophisticated investment strategies are likely to compete for liquidity at the same time.[5]

Formulating an effective rebalancing strategy for portfolios with substantial private asset allocations necessitates an understanding of their inherent illiquidity. Overlooking this illiquid characteristic in the rebalancing process can introduce adverse effects. The inability to maintain a stable risk and leverage profile might not align with a portfolio's predefined risk appetite. Furthermore, fluctuating levels of leverage amplifies complexities in collateral and balance sheet management and could affect other active strategies in a portfolio. While many previous studies have aimed to optimize portfolios containing both public and private assets by acknowledging the illiquid nature of private assets, their primary focus is on determining the optimal asset allocation, rather than on managing the portfolio (e.g., rebalancing), especially with regard to the portfolio's liquidity position through market cycles. These considerations highlight the criticality of meticulous planning and risk assessment in

[1] `https://globalswf.com/reports/2022annual/`

[2] `https://equable.org/pension-funding-trends-2023/`

[3] Information can be obtained from annual reports.

[4] The 'denominator effect' occurs when the values of publicly traded assets decrease while private asset values remain steady, possibly due to delayed or stale valuations. This results in a higher proportion of private assets within the overall portfolio and, consequently, an increase in the portfolio's weighting toward private assets. To further complicate the issue, while it is commonly known that the valuations of private assets are not necessarily up-to-date, they are used to determine the official market value for a fund, which in turn affects the rebalancing amount for public assets. In other words, the rebalancing amount of public assets are affected by stale valuations of private assets. This issue could be exacerbated for investors who invest in foreign countries, when the values of private assets rise due to currency effects coupled with stale valuations.

[5] Liquidity management is paramount for large, sophisticated institutional investors. These investors make extensive use of derivatives and absolute return strategies involving long and short positions, all of which require liquidity to support. As a result, many of these funds have advanced treasury and balance sheet management functions that are tasked with ensuring the portfolio's balance sheet can generate sufficient liquidity to meet various liquidity and cash needs.

the rebalancing process for portfolios invested in both public and private assets.

The goal of this study is to develop a solution for institutional investors, facilitating the management of portfolios that include private assets, particularly during market downturns. The objective is to achieve more stable profiles in terms of leverage, risk, and liquidity. This is achieved by designing a rebalancing strategy that holistically considers the underlying factor exposures, rather than asset class exposures, of the overall target portfolio allocation. This strategy uses public asset classes to complement the factor exposures of the existing private assets, instead of rigidly rebalancing these public assets to fixed asset allocations. The result is more stable profiles for the portfolio's leverage and risk, enabling the portfolio to maintain a better liquidity position.

In our study, we develop and compare two approaches of rebalancing investment portfolios that include private assets with the Traditional rebalancing strategy, which periodically readjusts the portfolio to its initial fixed-weight target. Our research places limitations on the ability to rebalance private assets, reflecting real-world constraints. However, we allow for full rebalancing of the public assets based on the rules established by the respective rebalancing strategies.

The first approach we consider, labelled as the Beta rebalancing strategy, utilizes a three-factor model comprising Equity, Real Rate, and Inflation factors to forecast the values of private assets. This approach differs from the Traditional strategy, which relies on the official valuations of private assets –often stale for months– to determine rebalancing amounts. The Beta rebalancing strategy calculates the rebalancing amount based on the forecasted values of these private assets using the factor model. Our objective is to assess whether using forecasted valuations for private assets, rather than relying on their stale valuations, can enhance the rebalancing methodology and thereby improve the management of a portfolio.

The second approach we develop, labelled as the Factor rebalancing strategy, expands upon the Beta rebalancing strategy. In addition to forecasting the value of private assets, it also uses a factor model to ensure that the factor exposures of the rebalanced portfolio match those of the initial target portfolio (i.e., the target asset allocation).[6] The Factor rebalancing strategy, instead of restoring public assets to their fixed weight targets, considers them as complementary components of the overall portfolio, effectively compensating for under or overweights in private assets by adjusting public asset allocations to achieve the desired factor allocation for the entire portfolio. Our aim is to evaluate the effectiveness of combining forecasted valuations for private assets with the management of a portfolio's factor exposures in maintaining desired characteristics, such as leverage, risk, and liquidity.

We conducted a historical analysis on the three rebalancing strategies applied to a representative portfolio held by sovereign wealth funds or pension funds, which includes nominal and real bonds, private and public equity, and real estate. The Traditional rebalancing strategy shows high variability in the levels of leverage, risk, and liquidity. Specifically, dur-

[6]For the Factor rebalancing strategy, the target allocation for the private assets remain unchanged.

ing the 2008 Global Financial Crisis and the Crash of 2022, the portfolio encounters liquidity issues. This is due to the forced rebalancing of public assets back to their target weights with the use of leverage when the private assets become overweight due to the 'denominator effect'.

The Beta rebalancing strategy partially tackles the challenge of stale private asset valuations by relying on model-based valuations for rebalancing decisions. In our analysis, we find that although there are slight improvements between valuation updates (e.g., annually) for private assets, this strategy doesn't deliver sustained benefits over multiple years because the forecasted values of private assets align with their observed values upon valuation updates. As a result, this approach may be better suited for investors who adjust their asset allocation annually and fine-tune their allocation to public assets based on the prevailing allocation to private assets.

The Factor rebalancing strategy stands out in managing portfolio leverage and risks over a long time frame, particularly in comparison to Traditional and Beta strategies. This approach effectively uses public assets (e.g., stocks and bonds) as complementary assets to the portfolio, dynamically adjusting their weights to align the portfolio's factor exposures with those of the initial target portfolio. When private assets are overweight and provide a higher Equity factor exposure than planned, the Factor rebalancing strategy compensates by reducing the need for public equities (i.e., S&P 500). In practice, this type of strategy is necessary to manage the fund's risk exposure because the weights of private assets cannot be readily rebalanced to target levels. By using public assets to complement the private assets, this strategy can better maintain the leverage and risk profiles of the portfolio.

This strategy not only provides better control over leverage and volatility but also better safeguard the portfolio's liquidity position. This is mainly attributed to the reduced use of leverage, allowing the portfolio to hold more unencumbered, high-quality liquid assets such as bonds. This aspect is particularly vital during the 2008 Global Financial Crisis and the Crash of 2022, as our analysis demonstrates that the Factor rebalancing strategy is most effective in preserving liquidity compared to the other two strategies. The significance of liquidity management is critical. Leveraged portfolios can face severe liquidity issues, as exemplified by the UK pension crisis in 2022.[7] Strategies that inadvertently increase leverage can exacerbate these liquidity challenges. The Factor rebalancing strategy can reduce the demand for liquidity for rebalancing purposes.

In summary, our study highlights the importance of reassessing the roles of public assets within portfolios that also include private assets, particularly in terms of maintaining the desired portfolio characteristics (e.g., exposures, leverage, risk, and liquidity) over time. This becomes crucial when considering the difficulties associated with rebalancing private assets. We suggest that public assets should be seen as complementary to private assets in an overall portfolio, allowing for flexible reallocation to compensate for the illiquid charac-

[7] The UK pension crisis in 2022 was exacerbated by the widespread use of leverage in Liability-Driven Investment (LDI) strategies. When bond yields rose sharply, these leveraged positions required substantial collateral, leading to liquidity challenges for pension funds. To meet these demands, many funds had to sell gilts, which further depressed prices and worsened the crisis.

teristics of the private assets. By adopting this approach, investors can gain better control over the portfolio's risk profile and enhance overall risk and liquidity management.

LITERATURE REVIEW

There is a growing literature on rebalancing. Generally, there are two types of strategy: periodic (based on fixed intervals) and threshold-based (triggered by deviations based on selected metrics, such as portfolio weights or risks, etc.). The choice depends on factors like investor preferences, portfolio size, and transaction costs (Masters, 2003; Israelov and Tummala, 2018; Tokat and Wicas, 2007; Sun et al., 2006; Ilmanen and Maloney, 2015; Cuthbertson et al., 2016). While studies, such as those by Dichtl et al. (2014) and Ilmanen and Maloney (2015), suggest that the choice of a rebalancing strategy has minimal effect on portfolios with public securities. There have been numerous studies on the effectiveness of rebalancing versus buy-and-hold strategies. Dichtl et al. (2014) compare buy-and-hold strategies with rebalancing strategies, revealing that rebalancing strategies perform better in volatile, trendless markets. Furthermore, rebalancing strategies can introduce negative convexity, potentially affecting portfolio performance (Rattray et al., 2020; Israelov and Tummala, 2018).

Various methods have been suggested for incorporating an illiquid asset into a portfolio. Some focus on risk management, estimating the likelihood of the illiquid asset crossing a certain threshold, as seen in the works of Milevsky (2004) and Wang and Peterson (2019). Others, such as Hayes et al. (2015), apply a penalty for illiquidity during portfolio optimization. Similarly, Kinlaw et al. (2013) conceptualize illiquidity as a shadow allocation within the portfolio. Aliaga-Diaz et al. (2022) provide a framework that incorporates the private equity into multiasset portfolio with the objective of maximizing an investor's utility of terminal wealth, while considering the uncertain cash flows. Ma and Pirone (2014), Baxter (2018), Ang et al. (2014), and Rudin et al. (2019) propose that for illiquid assets, a buy-and-hold strategy might be more suitable than the commonly assumed perfect rebalancing in many asset allocation models. In the buy-and-hold strategy, the optimal investment proportions are set at the start and allowed to shift naturally over time. This approach contrasts with periodic rebalancing and can result in markedly different optimal asset weights.

Even if investors assume a buy-and-hold strategy for private assets, their exposures need to be managed over time nonetheless. The management of private equity investments hinges on capital calls and distributions. Research in this field is twofold: one strand focuses on simulating these cash flows (Takahashi and Alexander, 2002; De Malherbe, 2005; Buchner et al., 2010; Bollen and Sensoy, 2022; O'Shea and Jeet, 2018). The other looks at strategies for balancing cash flows across vintages to maintain a target private equity allocation (Oberli, 2015; Shen et al., 2021).

Rebalancing the public assets within a portfolio that contains both public and private assets presents challenges. As emphasize by Conner (2003) and Couts et al. (2020), investors typically have access to reported (smoothed) private equity returns, but for accurate risk exposure and performance assessment, economic (unsmoothed) returns are necessary. This poses challenges in determining the valuation of private assets to be used in the rebalancing process because the valuation of private assets affects the fund's overall value, which, in turn, is used to determine the allocation to public assets.

DATA PREPARATION

Exhibit 1 describes the assets and factors used in this study. To simplify the explanation, we limited our analysis to just six asset classes. Our analysis includes four public asset classes in the U.S.: the S&P 500 representing public equities, U.S. Treasury Inflation-Protected Securities (TIPS) for inflation-linked government bonds, U.S. Treasuries (UST) as nominal government bonds, and the 3-month U.S. Treasury Bill symbolizing money markets (Cash). Our analysis includes two private asset classes in the U.S.: private equity and real estate. We represent Private Equity using a composite index, constructed from data in the Annual Financial Reports of U.S. state pension systems.[8] To represent Real Estate, we utilize the MSCI U.S. Quarterly Property Index. For our analysis, we use data from December 2001 to May 2023. We use three tradeable factors: Equity, Real Rate, and Inflation. Their definitions are provided in Exhibit 1. For easier interpretation in our analysis, all factors have been standardized to an annual volatility of 10%.

[Insert Exhibit 1 and Exhibit 2 here]

Panel A of Exhibit 2 shows results from linear regression analysis of historical asset returns with respect to factor returns.[9] Panel B of Exhibit 2 shows the results of factor loadings for private assets computed using the 2023 BNY Mellon's Capital Market Assumptions (CMAs) Report. As we will describe later, we use the factor loadings computed with historical data to forecast real-time observed returns for private assets, whereas we use the factor loadings computed from CMAs for rebalancing and risk estimation. As expected, the Equity factor loading computed from CMAs is higher for Private Equity, reflecting the practitioners' expectation that private equity investments are riskier than the volatility of their appraisal values would suggest.

METHODOLOGIES

In this section, we describe and provide an illustrative example for the three rebalancing strategies: Traditional, Beta, and Factor rebalancing. In this article, we assume that all three rebalancing strategies are triggered on a fixed schedule (e.g., monthly rebalancing).[10]

[8] This data source consists of a closed group with no selection biases. Initially, 94 investors were chosen, which was then reduced to 65 with the same fiscal year-end date for consistent measurement. Then, 53 state systems reporting private equity returns during the study period were filtered. Finally, 19 of them were chosen since they were consistently operated such portfolios for all fiscal years. The data are collected from the website linked below. `https://www.investmentcouncil.org/wp-content/uploads/2023/03/Cliffwater-Long-Term-PE-Performance46.pdf` (data accessed on November 20th 2023).

[9] We run this regression on annual returns given that the private equity returns are available at an annual frequency.

[10] However, in practice, practitioners may choose to trigger them based on different criteria.

The illustrative example is set up as follows. Initially, a portfolio is set up with predetermined target weights: 45% in the S&P 500, 10% in TIPS, 20% in UST, 12.5% in Private Equity, 12.5% in Real Estate, and 0% in Cash.[11] This portfolio, which is designed not to have leverage, has a Net Asset Value (NAV) of $100.

From the beginning (time 0) to the first rebalancing period (time 1), the Equity, Real Rate, and Inflation Factor experience returns of -30%, -15%, and 5%, respectively. Based on the factor loadings in Panel A in Exhibit 2, these factor returns correspond to asset returns of -45.9%, -17.4%, and -20.85% for the S&P 500, TIPS, and UST, respectively. The values of private assets (Private Equity and Real Estate) remain unchanged during this period, reflecting stagnant valuations for these private assets. At time 1, the portfolio is rebalanced in accordance with each strategy. Moving forward, from time 1 to time 2, Private Equity and Real Estate are revalued, showing returns of -12% and -7% respectively. This change reflects the infrequent updates in the valuations of private assets, typically performed on a quarterly or annual basis. The portfolio undergoes another rebalancing at time 2, following respective strategies.

During each rebalancing period, only the public assets (S&P 500, TIPS, and UST) are adjusted, while no transactions occur for private assets (Private Equity and Real Estate) in this illustration. This choice is designed to replicate the real-world scenario in which private assets are typically excluded from the rebalancing process. In the example, the pre- and post-rebalancing dollar amounts and portfolio weights for the six assets are reported, along with the transaction amounts.

Traditional rebalancing strategy

The strategy described here is a traditional method for rebalancing a portfolio to its target allocation, as detailed in Exhibit 3. Initially, at time 1, the S&P 500, TIPS, and UST are rebalanced to their target weights. Overweight assets (TIPS and UST) are sold, and the underweight asset (S&P 500) is purchased. However, private assets are not rebalanced and remain off target. Since the proceeds from the sales of TIPS and UST are insufficient to purchase the necessary amount of the S&P 500, leverage is used to bring its weight back to the 45% target. This use of leverage results in a Cash weight of -9.0%. At time 2, when the valuations of private assets are revised lower, the portfolio's NAV decreases. This decrease results in the S&P 500, TIPS, and UST becoming overweight in terms of percentages of NAV, leading to their sale during rebalancing. After completing the rebalancing process, the portfolio remains leveraged with a weight of -6.8% for the Cash asset class.

[Insert Exhibit 3 here]

[11] This allocation is representative of sovereign wealth funds, as reported by the Global SWF 2022 annual report.

Beta rebalancing strategy

The Beta rebalancing strategy bears resemblance to the Traditional rebalancing strategy but differs in its use of forecasted values for private assets and the corresponding projected portfolio NAV instead of using their observed values. The results of applying the Beta rebalancing method to a portfolio are illustrated in Exhibit 4. This strategy assumes that ultimately, the observed valuations of private assets are used to determine the portfolio's NAV, which in turn determines the corresponding allocation to public assets.

At time 1, the model forecasts a return of -15.3% and -9.3% for Private Equity and Real Estate, respectively.[12] The model calculates the return r_m for asset i using the equation

$$r_{m,i} = c + L'_{h,i} r_f \tag{1}$$

where $L_{h,i}$ represents the historical factor loadings vector for asset i reported in Panel A of Exhibit 2, r_f is the vector of factor returns, and c is the constant term.

For rebalancing purposes, these forecasted asset values are utilized, although the portfolio's official financial metric still relies on observed valuations. When assessed through observed valuations, all public assets appear underweight, as shown by 'Δw − *Actual*' in Exhibit 4. Under the Beta rebalancing strategy, these underweights in public assets reflect an expectation that real-time portfolio values are lower than reported.

[Insert Exhibit 4 here]

At time 2, when private asset valuations are updated, the Beta rebalancing aligns with the Traditional approach, resulting in identical portfolio compositions. However, the transaction amounts differ due to the distinct portfolio compositions at time 1. During rebalancing periods with stagnant private asset valuations, the Beta rebalancing approach suggests for less portfolio leverage in a declining market, acknowledging the moderate to significant correlation of most private asset classes with equities.

Factor rebalancing strategy

The Factor rebalancing strategy extends the Beta rebalancing strategy further. It uses a factor model to also calculate the portfolio's factor exposures, albeit with different factor loadings that better reflect the risk characteristics of private assets. The Factor rebalancing strategy aims to align the portfolio with its initial factor exposures, rather than its initial target asset weights. This alignment is achieved by adjusting the weights of the public assets to ensure the rebalanced portfolio has the desired factor exposures (i.e., those implied by the target portfolio).

This illustration employs a three-factor model and three public assets, facilitating for

[12] Calculated using the factor loadings from Panel A of Exhibit 2 and factor returns r_f at time 1. The constant term (c) used for private equity is 11.3%/12 = 0.94%, as we assume this change happens over 1 month. The constant term for real estate is 0.6%.

straightforward computations to arrive at unique portfolio weights for achieving the desired factor exposure.[13] The vector of portfolio factor exposures is calculated using the equation

$$b_f = L_m w_a \tag{2}$$

where b_f is the vector of factor exposures, L_m is the matrix of factor loadings reported in Panel B of Exhibit 2, and w_a is the vector of asset weights.

Results using the Factor rebalancing strategy are illustrated in Exhibit 5 and Exhibit 6. Initially, based on the target weights set for the portfolio, the factor exposures for the Equity, Real Rate, and Inflation factors were 110.4%, 36.9%, and -22.0%, respectively. By the time we reach time 1, the portfolio allocation experiences a drift.[14] During rebalancing, the weights for the S&P 500, TIPS, and UST are adjusted to achieve the portfolio's factor exposures that match the initial factor exposure, as calculated by Equation (2). The private assets are not rebalanced. The objective is to find w_a^* such that the following expression is satisfied

$$L_m w_a^* - b_{f,initial} = 0 \tag{3}$$

where $b_{f,initial}$ is the initial portfolio factor exposures and the elements of w_a^* corresponding to private equity and real estate are fixed. Exhibit 6 shows that post-rebalancing, the portfolio's factor exposures are restored to their initial values when forecasted NAV is used.[15] At time 2, the same procedure repeats after the valuation for the private assets is updated.

The Factor rebalancing strategy, while unable to rebalance the private assets, strategically underweights the public assets, especially the S&P 500 (40.8% vs 45%), to offset the overweight in the *factor exposures* of the private assets. This approach balances the factor exposures and enables the portfolio to maintain its initial factor exposures. Additionally, this underweighting of public assets reduces the use of leverage. In this example, the portfolio has lower leverage, with a Cash weight of -2.6%, in contrast to the -6.8% leverage employed by the Traditional and Beta rebalancing strategies.

While it is not ideal to forego rebalancing private assets for practical reasons, the Factor rebalancing strategy is designed to address this real-world constraint by keeping the initial factor exposures intact through adjustments to public assets. This intentional preservation of the initial factor exposures helps the portfolio maintain a risk level that is similar to its original, a critical consideration when the portfolio adheres to a predefined risk tolerance.

[13] In more complex real-world scenarios with more assets available for rebalancing than the number of factors in the model, there can be many portfolios with the same set of factor exposures. Advanced methodologies become necessary. These methodologies help in forming a portfolio that effectively meets the required factor exposures, as elaborated in research such as that by Elkamhi, Lee, and Salerno (2021) or Lee and Salerno (2023).

[14] See 'Time 1 - before rebalance' for the Factor rebalancing strategy on the column set 'Using Model NAV'.

[15] For ease of comparisons, Exhibit 6 shows the factor exposures for all three strategies with both the observed (Actual) and forecasted (Model) NAV.

Practical considerations for the factor model

The first purpose of using a factor model is to forecast the observed valuation returns of private assets, which reflect the characteristics of appraisal valuations. To achieve this, we rely on historical returns to directly estimate the factor loadings for private assets. In practice, practitioners select their preferred factor model and estimate these factor loadings using data that offers the most accurate forecast of observed returns for their privately owned assets. It is essential to choose the appropriate dataset for model estimation because the goal here is to forecast changes in the appraisal value of private assets.

The second purpose of our factor model is to determine the factor exposures of the various assets in the portfolio and calculate the necessary rebalancing amounts. To achieve this, investors choose a model that best reflects the assets' exposure and risk characteristics, such as those indicated by their asset allocation or risk management models. In this context, the factor exposures of private assets may appear to be larger in magnitude compared to those estimated using realized performance. This discrepancy arises because many practitioners acknowledge that valuation returns for private assets often lag and have a smoother pattern relative to what many believe to be their true underlying characteristics (Conner, 2003; Couts et al., 2020).[16]

A HISTORICAL ILLUSTRATION

Setup

In our historical analysis, we examined the three rebalancing strategies –Traditional, Beta, and Factor– spanning from December 2001 to May 2023. The Target portfolio has the following weights: 45% in S&P 500, 10% in TIPS, 20% in U.S. Treasuries, 12.5% in Private Equity and 12.5% in Real Estate.

To account for the low liquidity of Private Equity and Real Estate, we permit these assets to be partially rebalanced at 5% of the amount required for a full rebalance back to their initial target asset weights. This simplified modelling approach mirrors the real-world characteristics of private assets, which are not regularly rebalanced, allowing their portfolio weights to drift from their target allocations over time. However, by allowing for a small monthly fraction of transactions, it prevents our historical simulation from excessively over- or under-weighting these private assets as time passes. This reflects the real-world practices of asset managers who manage them over longer time frames and not month over month. For example, investors can manage the cash flows over time to achieve a certain target allocation, as studied by Oberli (2015) and Shen et al. (2021).

[16] For example, the private equity composite index used in this study exhibits an annual volatility of approximately 16%, which is similar to that of public equities. However, in many asset allocation and risk models, the risk associated with private equity is typically modeled as higher. Capital Market Assumptions produced by the asset management industry often indicate an annualized volatility exceeding 20% for private equity. To illustrate, the 2023 Survey of Capital Market Assumptions by Horizon Actuarial Services reports an annualized volatility of 22.57% for the private equity asset class based on a survey of 42 investment advisors. BlackRock goes even further with an assumption of 32% volatility for U.S. private equity buyout funds.

[Insert Exhibit 7 here]

Every month, we rebalance fully the public assets and partially the private assets, based on the three rebalancing strategies. Private asset valuations are updated annually in June, utilizing the annual returns acquired from the annual financial reports, as previously explained.[17] Exhibit 7 shows a horizontal timeline representing a schedule for the rebalancing in our historical analysis. The timeline is color-coded with different events marked in specific colors.

Metrics

To assess the effectiveness of the three rebalancing strategies, we analyze several key metrics. Firstly, we compare the turnovers associated with each strategy. Then, we track and examine the evolution of portfolio weights over time. We study the use of leverage in more detail and assess its stability, especially during the 2008 Global Financial Crisis (GFC) and the Crash of 2022. Additionally, we examine portfolio risk (volatility) over time, as computed using the formula

$$\sigma_{t,p} = \sqrt{w_t' \Sigma w_t} \tag{4}$$

Here, $\sigma_{t,p}$ represents the calculated portfolio risk at time t, w_t denotes the portfolio weights for various assets at time t, and Σ is the covariance matrix derived from the 2023 BNY Mellon's CMAs Report.[18] We also assess the stability of this risk metric over time and its behavior during the 2008 GFC and Crash of 2022. Lastly, we evaluate the Liquidity Coverage Ratio (LCR) over time, its stability, and its performance during the crisis periods.

Liquidity Coverage Ratio (LCR). The LCR functions as a tool to assess a portfolio's ability to fulfill its short-term obligations, which is a vital consideration for investors worried about liquidity risk. It aims to calculate the proportion between (a) the amount of unencumbered high-quality liquid assets (HQLA) readily convertible into cash through sales or repurchase agreements or available to be used as collateral for meeting margin requirements and (b) the potential short-term liquidity requirements during periods of financial stress, usually spanning 30 to 90 days.[19] The exact formulation of the LCR may vary between institutions; however, for the purposes of this illustration, we use the following simple formula:

$$LCR = \frac{\text{Stock of unencumbered High-Quality Liquid Assets (HQLA)}}{\text{Total Net Cash Outflows over the stress period}} \tag{5}$$

The unencumbered High-Quality Liquid Assets (HQLA) include assets that are con-

[17] For simplicity, we make the assumption that private assets bought or sold between July and May of the following year are transacted at the valuation from the preceding June.

[18] Since this is an illustration, and covariance matrices from CMAs reports reflect stable long-term risk characteristics, we choose to use the most recent CMAs report from BNY Mellon for simplicity.

[19] Short-term potential liquidity needs are evaluated through scenario or Monte Carlo simulation analyses. These analyses help assess the cash or collateral requirements that could emerge during stressful market events to cover cashflow needs or margin requirements.

sidered sources of liquidity by market participants.[20] These assets typically include cash and highly-rated government securities, among others. Since some of these assets can decrease in value during a stress scenario (for example, government bonds may lose value if interest rates rise), financial institutions commonly apply a haircut to assess the value of collateral. In our analysis, we compute the unencumbered HQLA as the sum of cash and 70% of the total market value of TIPS and UST.[21] When the Cash weight is negative, it has the effect of decreasing the High-Quality Liquid Assets (HQLA).[22]

In our analysis, we simplify this estimation by using 10% of the Net Asset Value (NAV) as the Total Net Cash Outflows. We assume that the asset owners of our portfolios may need access to a 10% liquidity reserve during periods of stress to cover liquidity needs.[23] A LCR below 1 indicates an elevated liquidity risk, implying that the portfolio may not have enough liquid assets to fulfill its liquidity obligations in the event of a market stress.

Results and Discussion

Turnover. Exhibit 8 reports on the turnover rates for public assets (S&P 500, TIPS, UST, and Cash), with turnover defined as the standard deviation of the monthly rebalancing amounts.[24] An analysis of the turnovers for the S&P 500, TIPS, and UST shows no significant differences in rebalancing amounts across the three examined strategies, indicating no material transaction cost savings between them. However, the situation with Cash is different. Cash undergoes monthly rebalancing to varying amounts due to changes in the portfolio's leverage. Specifically, an increase in leverage would occur if additional cash is required for rebalancing activities.

[Insert Exhibit 8 here]

An important finding is that the Traditional rebalancing strategy experiences the highest turnover for Cash, with a standard deviation of 0.97% per month. This implies that the portfolio's leverage is highly variable under this strategy. In contrast, the Factor rebalancing strategy demonstrates the lowest Cash turnover (0.24% per month), indicating a relatively stable portfolio leverage. The Beta rebalancing strategy yields results between the other two strategies. In a next section, We discuss how a stable leverage profile contributes to more steady portfolio risk and Liquidity Coverage Ratio (LCR) profiles for the Factor rebalancing

[20]HQLA can be converted into cash through repurchase agreements, pledged as collateral, or easily liquidated.

[21]The 70% haircut represents an illustrative but conservative assumption for the potential loss in value during a stress event. This large haircut can represent losses in stress markets for bonds and overcollateralization.

[22]In practical terms, a negative Cash weight indicates borrowing, typically achieved through a repurchase agreement where Bonds are used as collateral. Consequently, the portfolio's quantity of unencumbered Bonds decreases, leading to a reduction in HQLA. This reduction occurs because when we subtract the borrowed Cash amount from the amount of Bonds, it signifies that some of those Bonds have been pledged as collateral, thereby reducing the quantity of unencumbered Bonds and ultimately lowering the Liquidity Coverage Ratio (LCR).

[23]Total Net Cash Outflows are typically estimated by taking into account factors like capital withdrawals, drawdowns on committed credit and liquidity facilities, along with other contractual and contingent outflow obligations from active investment strategies. The estimation of contingent outflow during stress events may employ a Monte Carlo analysis and proper netting of counterparties.

[24]Notably, private assets are excluded from this report as their rebalancing amounts are minimal.

strategy.

Portfolio Weights and Leverage. Panels A to C of Exhibit 9 visually depict the time-evolving portfolio weights of Private Equity, S&P 500, Real Estate, U.S. Treasuries (UST), and Treasury Inflation-Protected Securities (TIPS). The sum of these weights at any point constitutes the portfolio's gross asset weight, also detailed in Panel D. Gross asset weight represents the total economic exposure of the portfolio, and a value above 100% signifies leverage use in the portfolio.[25] Panel A of Exhibit 10 presents the 5th, 50th, and 95th percentiles of portfolio weights across the historical period. Panel A of Exhibit 11 reports how the portfolio weights, especially the weights for Cash, change during the 2008 GFC and the Crash of 2022.

[Insert Exhibit 9 here]

Examining Panel A to D of Exhibit 9 and Panel A of Exhibits 10 and 11 reveals that the Traditional and Beta rebalancing strategies exhibit high variation in gross asset weights. Despite the median leverage being low across all strategies (evident from the small, negative median Cash weights), the 5th percentile Cash weights in both Traditional and Beta rebalancing strategies are significantly more negative compared to the Factor rebalancing strategy. While the Beta rebalancing strategy aims to mitigate the impact of stale valuations and the 'denominator effect' in private assets by incorporating model-forecasted returns, it is only effective during periods between valuation updates. The Factor rebalancing strategy maintains the most stable gross asset weight over time, particularly during the 2008 Global Financial Crisis (GFC) and the Crash of 2022.

[Insert Exhibit 10 here]

Panel A of Exhibit 11 provides a view of portfolio weights during the peak moments of the 2008 GFC and the Crash of 2022. Analyzing the results reveals a significant impact of the Traditional rebalancing strategy on portfolio leverage during the Crash of 2022. The total weight allocated to risk assets, which include Private Equity, S&P 500, and Real Estate, surges from 70.4% to 78.6%. The Cash allocation plunges from -0.4% to -8.6%. However, the Factor rebalancing strategy exhibits a smaller change in leverage, with the Cash allocation changing from 1.4% to -1.9%. The Factor strategy lowers the weights assigned to the S&P 500 (from 44.0% to 39.8%), while the weights for bonds remains roughly unchanged. These adjustments counteract the changes in factor exposures of the portfolio due to the 'denominator effect' of private assets. In the 2008 GFC, the Factor rebalancing strategy underweights both the S&P 500 and bonds.

[Insert Exhibit 11 here]

[25] For instance, a 112% gross asset weight implies a -12% cash weight (e.g., 12% of the portfolio NAV is financed).

Portfolio Risk (Volatility). Panel E of Exhibit 9 visually demonstrates how portfolio risk varies over time. It's evident that portfolio risk is significantly influenced by the gross asset weight. When the portfolio has a higher gross asset weight (indicating higher leverage), its risk generally increases given this is an equity-centric portfolio. This effect is particularly pronounced in the case of Traditional and Beta rebalancing strategies, where the portfolios become heavily leveraged during the Global Financial Crisis (GFC) and the Crash of 2022, leading to sharp rises in portfolio risk. This situation can be problematic if the portfolio must adhere to strict risk limits. The performance of the Beta rebalancing strategy indicates that considering only the stale valuation of private assets is insufficient for maintaining a stable portfolio risk level.

The Factor rebalancing strategy exhibits significantly more stable portfolio risk over time. During both crises, the portfolio's risk remains more stable. This can be attributed to the fact that the Factor rebalancing strategy maintains the portfolio's factor exposures at their initial values and thereby preserves the level of major risk exposures of the portfolio.

As shown in Panel B of Exhibit 11, specifically between November 2021 and September 2022, the portfolio risk for the Traditional and Beta rebalancing strategies rise from 12.2% to 13.1%, and from 12.0% to 12.8%, respectively. Meanwhile, the portfolio risk for the Factor rebalancing strategy shows a minimal change (i.e., from 11.8% to 11.9%). Panel B of Exhibit 10 shows that the medians of portfolio risk over the entire sample period are quite similar across the three rebalancing strategies. However, the Factor rebalancing strategy exhibits higher stability in portfolio risk, with a spread of only 0.2% between the 5th and 95th percentile values, compared to a 0.8% to 1.0% spread in the other two strategies. In other words, employing the Factor rebalancing strategy would result in a more stable portfolio risk profile during this sample period.

Liquidity Coverage Ratio (LCR). Panel F of Exhibit 9 illustrates how the Liquidity Coverage Ratio (LCR) changes over time. The observations regarding LCR align with those related to leverage and portfolio risk. As the portfolio's leverage increases (e.g., with a rise in the gross asset weight), the Cash weight turns negative to reflect the need for financing. This heightened borrowing to facilitate rebalancing leads to a reduction in High-Quality Liquid Assets (HQLA), ultimately resulting in a decrease in the LCR.[26]

The decline in LCR is pronounced for the Traditional and Beta rebalancing strategies. During the 2008 GFC and the Crash of 2022, as the portfolios experience losses in market value, the rebalancing algorithms utilize leverage to rebalance the portfolios and bring all public assets back to their target weights. This puts significant stress on the portfolios' liquidity position. During both crises, the LCR under both strategies experience significant deterioration, approaching a value of 1. If this example is representative, it would likely necessitate a change in the portfolio's target composition and/or a plan to sell private assets quickly, as asset owners would likely not tolerate the LCR being below 1.0.

Analyzing the Factor rebalancing strategy reveals a more effective risk and liquidity

[26]This is because our Total Net Cash Outflows during the stress period remain at 10%.

management approach. As the market enters a crisis, the fund's leverage remains relatively stable. The decline in LCR is primarily due to the market value loss of bond portfolios, without additional stress factors such as increased leverage. This leads to less deterioration of the LCR metric, enabling the portfolio to maintain a relatively healthy level of liquidity even during a crisis.

Examining Panel B of Exhibits 10 and 11 further highlights the effectiveness of the Factor rebalancing strategy in managing the Liquidity Coverage Ratio (LCR). For example, between November 2021 and September 2022, the portfolio employing the Factor rebalancing strategy experiences a relatively mild decrease in LCR, going from 2.29 to 1.94. In contrast, the Traditional rebalancing strategy sees a substantial drop in LCR, plummeting from 2.06 to 1.24 during the same period. Similarly, the Beta rebalancing strategy witnesses a decline in LCR, falling from 2.13 to 1.38. The Factor rebalancing strategy demonstrates its ability to (1) maintain a higher level of liquidity and (2) better mitigate the adverse impact on liquidity during a crisis event.

CONCLUSION

In summary, this study develops and examines two rebalancing strategies –Beta rebalancing and Factor rebalancing strategies–designed for portfolios that include private assets, and compare them with a traditional rebalancing strategy. The Traditional rebalancing strategy, while effective for portfolios consisting solely of public assets, proves inadequate when dealing with portfolios containing private assets. This inadequacy stems from the inability to adapt to the unique characteristics of private assets, including their illiquidity and infrequent valuation updates. When public assets are required to rebalance to their target weights, it can lead to adverse effects on the portfolio, such as an increase in leverage use during a crisis due to the 'denominator effect', fluctuations in portfolio risks, and a deterioration in the portfolio's liquidity position during stressful events.

The two rebalancing strategies examined in this paper address the presence of private assets. Specifically, the Beta rebalancing strategy utilizes a factor model to estimate changes in private asset values and adjusts the portfolio accordingly. The Factor rebalancing strategy builds upon the Beta rebalancing approach by aligning the portfolio with the factor exposures of the initial target portfolio, ensuring the intended leverage, risk, and liquidity profile are maintained. We illustrate that while the Beta rebalancing strategy can navigate the portfolio between private assets' valuations, it proves inadequate in managing portfolio leverage, risk, and liquidity over time. The results for the Beta strategy generally resemble those of the Traditional strategy.

Empirical analysis shows the effectiveness of the Factor rebalancing strategy. By aligning the portfolio's underlying factor exposures with their initial levels, this rebalancing process leads to more stable levels of leverage over time. This results in a more stable portfolio volatility profile, which is advantageous for risk management purposes and enhances adherence to the portfolio's risk appetite. Importantly, the Factor rebalancing strategy enables the portfolio to maintain healthy liquidity levels, even during a market crisis. The deterioration of liquidity is more manageable and potentially less catastrophic when compared to the other two methods. Given the significant and increasing allocation to private assets among large institutional investors, to effectively manage such portfolio through economic cycles, reevaluating the portfolio's rebalancing strategy holds strategic importance.

In conclusion, the incorporation of the Beta and Factor rebalancing strategies in portfolios containing private assets has been demonstrated to offer distinct advantages over the Traditional rebalancing strategy. These advantages primarily manifest in better management of portfolio leverage, risk, and liquidity, particularly in the context of the unique challenges posed by private assets. However, an intriguing avenue for future research would be to explore how these rebalancing strategies might influence the portfolio optimization decision, particularly in terms of asset allocation weights for private assets. Such an investigation could reveal deeper insights into the dynamic interplay between asset allocation and rebalancing strategies, potentially leading to more refined and effective portfolio management approaches for institutional investors with significant private asset holdings.

DISCLAIMER

The views and opinions expressed in this study are solely those of the authors and do not necessarily reflect the official policy or position of any other agency, organization, employer, or company, including the authors' employer in the asset management industry. The information provided in this study is for general informational purposes only and should not be construed as legal, tax, investment, financial, or other professional advice.

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Exhibit 1: The Assets and Factors

Asset	**Description**
S&P500	Total return of the S&P 500 Index
TIPS	Total return of the S&P 15+ Year US Treasury TIPS Index
UST	Total return of the iBoxx USD Treasuries 15Y+ Index
Private Equity	Total return of the Composite Private Equity Index
Real Estate	Total return of the MSCI U.S. Quarterly Property Index
Cash	Total return of 3-month U.S. Treasury Bills (rolled monthly)

Factors	**Description**
Equity	Total return of the S&P 500 Index
Real Rate	Total return of the S&P 15+ Year US Treasury TIPS Index
Inflation	Total return of a portfolio that is long 1 unit of TIPS and short 1 unit of UST

All Factors are normalized to have 10% annualized volatility

This exhibits provides a description of the assets and the tradeable macroeconomic factors used in the analysis. The Composite Private Equity Index is constructed from data provided in Annual Financial Reports published by the U.S. state pension systems. The combination of long and short positions in the Inflation factor is designed to represent a break-even position on inflation. For easier interpretation in our analysis, all factors have been standardized to an annual volatility of 10%.

Exhibit 2: Factor Loadings (Betas) of Assets

	S&P500	TIPS	UST	Private Equity	Real Estate
Panel A: Factor Loadings & Regression constants: Historical data					
Equity Factor	1.53	0.00	0.00	0.75	0.40
Real Rate Factor	0.00	1.16	1.16	-0.27	-0.17
Inflation Factor	0.00	0.00	-0.69	0.49	-0.09
Constant (Annualized)	*0.0%*	*0.0%*	*0.0%*	*11.3%*	*7.2%*
Panel B: Factor Loadings: Capital Market Assumptions					
Equity Factor	1.78	0.00	0.00	2.06	0.36
Real Rate Factor	0.00	1.24	1.24	-0.11	0.09
Inflation Factor	0.00	0.00	-0.94	-0.14	-0.11

This exhibit shows the regression coefficients (i.e., factor loadings) of the assets with respect to the factors. Panel A shows the factor loadings calculated using historical data. Panel B shows the factor loadings of private assets computed using the 2023 Capital Market Assumptions report from BNY Mellon.

Exhibit 3: Example: Traditional rebalancing strategy

	S&P500	**TIPS**	**UST**	**Private Equity**	**Real Estate**	**Cash**	**Total**
Time 0: Starting values							
Net Asset Value (NAV)	\$45.00	\$10.00	\$20.00	\$12.50	\$12.50	\$0.00	\$100.00
Weights (w) - Target	45.0%	10.0%	20.0%	12.5%	12.5%	0.0%	100.0%
Time 1: Values of public assets change and are rebalanced. Values of private assets remain stale.							
Observed Return	-45.9%	-17.4%	-20.9%	0.0%	0.0%	0.0%	
NAV - before rebalance	\$24.35	\$8.26	\$15.83	\$12.50	\$12.50	\$0.00	\$73.44
w - before rebalance	33.2%	11.2%	21.6%	17.0%	17.0%	0.0%	100.0%
Rebalance Amount	*\$8.70*	*-\$0.92*	*-\$1.14*				
NAV - after rebalance	\$33.05	\$7.34	\$14.69	\$12.50	\$12.50	-\$6.64	\$73.44
w - after rebalance	45.0%	10.0%	20.0%	17.0%	17.0%	-9.0%	100.0%
Δw	*0.0%*	*0.0%*	*0.0%*	*4.5%*	*4.5%*	*-9.0%*	
Time 2: Values of private assets are updated.							
Observed Return	0.0%	0.0%	0.0%	-12.0%	-7.0%	0.0%	
NAV - before rebalance	\$33.05	\$7.34	\$14.69	\$11.00	\$11.63	-\$6.64	\$71.06
w - before rebalance	46.5%	10.3%	20.7%	15.5%	16.4%	-9.3%	100.0%
Rebalance Amount	*-\$1.07*	*-\$0.24*	*-\$0.48*				
NAV - after rebalance	\$31.98	\$7.11	\$14.21	\$11.00	\$11.63	-\$4.86	\$71.06
w - after rebalance	45.0%	10.0%	20.0%	15.5%	16.4%	-6.8%	100.0%
Δw	*0.0%*	*0.0%*	*0.0%*	*3.0%*	*3.9%*	*-6.8%*	

Δw is (w - after rebalance) minus (w - Target)

This table illustrates how the rebalancing method operates within the context of the example described in the METHODOLOGIES section. 'Observed Return' represents the observed return for the assets. These returns are applied to the assets' NAV to compute the 'NAV - before rebalance' amounts. 'w - before rebalance' is calculated by dividing 'NAV - before rebalance' by the Total NAV before rebalance. 'NAV - after rebalance' is determined by adding the rebalance amount to 'NAV - before rebalance'. Δw is the difference between the weight after rebalancing and the initial target weight for a given asset.

Exhibit 4: Example: Beta rebalancing strategy

	S&P500	**TIPS**	**UST**	**Private Equity**	**Real Estate**	**Cash**	**Total**
Time 0: Starting values							
Net Asset Value (NAV)	$45.00	$10.00	$20.00	$12.50	$12.50	$0.00	$100.00
Weights (w) - Target	45.0%	10.0%	20.0%	12.5%	12.5%	0.0%	100.0%
Time 1: Values of public assets change and are rebalanced. Values of private assets remain stale.							
Observed Return	-45.9%	-17.4%	-20.9%	0.0%	0.0%	0.0%	
Model Return	-45.9%	-17.4%	-20.9%	-15.3%	-9.3%	0.0%	
Model NAV - before rebalance	$24.35	$8.26	$15.83	$10.59	$11.34	$0.00	$70.37
Model w - before rebalance	34.6%	11.7%	22.5%	15.1%	16.1%	0.0%	100.0%
Rebalance Amount	*$7.32*	*-$1.22*	*-$1.76*				
Model NAV - after rebalance	$31.66	$7.04	$14.07	$10.59	$11.34	-$4.34	$70.37
Model w - after rebalance	45.0%	10.0%	20.0%	15.1%	16.1%	-6.2%	100.0%
Δw - *Model*	*0.0%*	*0.0%*	*0.0%*	*2.6%*	*3.6%*	*-6.2%*	
Actual NAV - after rebalance	$31.66	$7.04	$14.07	$12.50	$12.50	-$4.34	$73.44
Actual w - after rebalance	43.1%	9.6%	19.2%	17.0%	17.0%	-5.9%	100.0%
Δw - *Actual*	*-1.9%*	*-0.4%*	*-0.8%*	*4.5%*	*4.5%*	*-5.9%*	
Time 2: Values of private assets are updated. Model NAV is the same as Actual NAV.							
Observed Return	0.0%	0.0%	0.0%	-12.0%	-7.0%	0.0%	
NAV - before rebalance	$31.66	$7.04	$14.07	$11.00	$11.63	-$4.34	$71.06
w - before rebalance	44.6%	9.9%	19.8%	15.5%	16.4%	-6.1%	100.0%
Rebalance Amount	*$0.31*	*$0.07*	*$0.14*				
NAV - after rebalance	$31.98	$7.11	$14.21	$11.00	$11.63	-$4.86	$71.06
w - after rebalance	45.0%	10.0%	20.0%	15.5%	16.4%	-6.8%	100.0%
Δw	*0.0%*	*0.0%*	*0.0%*	*3.0%*	*3.9%*	*-6.8%*	

Δw *is (*w *- after rebalance) minus (*w *- Target)*

This table illustrates how the rebalancing method operates within the context of the example described in the METHODOLOGIES section. 'Observed Return' represents the observed return for the assets. 'Model Return' represents the factor model return for the private assets between official valuations. These returns are applied to the assets' NAV to compute the 'NAV - before rebalance' amounts. 'w - before rebalance' is calculated by dividing 'NAV - before rebalance' by the Total NAV before rebalance. 'NAV - after rebalance' is determined by adding the rebalance amounts to 'NAV - before rebalance'. Δw is the difference between the weight after rebalancing and the initial target weight for a given asset. At time 1, since the portfolio NAVs differ when using the observed and model returns, the portfolio weights based on the actual NAV are also provided for reference purposes. At time 2, the observed returns for private assets are applied to the time 0 observed asset values. After the values of the private assets are updated, the Model NAV is the same as the Actual NAV.

Exhibit 5: Example: Factor rebalancing strategy

	S&P500	TIPS	UST	Private Equity	Real Estate	Cash	Total
Time 0: Starting values							
Net Asset Value (NAV)	\$45.00	\$10.00	\$20.00	\$12.50	\$12.50	\$0.00	\$100.00
Weights (w) - Target	45.0%	10.0%	20.0%	12.5%	12.5%	0.0%	100.0%
Time 1: Values of public assets change and are rebalanced. Values of private assets remain stale.							
Observed Return	-45.9%	-17.4%	-20.9%	0.0%	0.0%	0.0%	
Model Return	-45.9%	-17.4%	-20.9%	-15.3%	-9.3%	0.0%	
Model NAV - before rebalance	\$24.35	\$8.26	\$15.83	\$10.59	\$11.34	\$0.00	\$70.37
Model w - before rebalance	34.6%	11.7%	22.5%	15.1%	16.1%	0.0%	100.0%
Rebalance Amount	*\$4.72*	*-\$0.68*	*-\$2.32*				
Model NAV - after rebalance	\$29.07	\$7.58	\$13.51	\$10.59	\$11.34	-\$1.73	\$70.37
Model w - after rebalance	41.3%	10.8%	19.2%	15.1%	16.1%	-2.5%	100.0%
Δw - *Model*	*-3.7%*	*0.8%*	*-0.8%*	*2.6%*	*3.6%*	*-2.5%*	
Actual NAV - after rebalance	\$29.07	\$7.58	\$13.51	\$12.50	\$12.50	-\$1.73	\$73.44
Actual w - after rebalance	39.6%	10.3%	18.4%	17.0%	17.0%	-2.4%	100.0%
Δw - *Actual*	*-5.4%*	*0.3%*	*-1.6%*	*4.5%*	*4.5%*	*-2.4%*	
Time 2: Values of private assets are updated. Model NAV is the same as Actual NAV.							
Observed Return	0.0%	0.0%	0.0%	-12.0%	-7.0%	0.0%	
NAV - before rebalance	\$29.07	\$7.58	\$13.51	\$11.00	\$11.63	-\$1.73	\$71.06
w - before rebalance	40.9%	10.7%	19.0%	15.5%	16.4%	-2.4%	100.0%
Rebalance Amount	*-\$0.10*	*\$0.16*	*\$0.07*				
NAV - after rebalance	\$28.97	\$7.74	\$13.58	\$11.00	\$11.63	-\$1.85	\$71.06
w - after rebalance	40.8%	10.9%	19.1%	15.5%	16.4%	-2.6%	100.0%
Δw	*-4.2%*	*0.9%*	*-0.9%*	*3.0%*	*3.9%*	*-2.6%*	

Δw is (w - after rebalance) minus (w - Target)

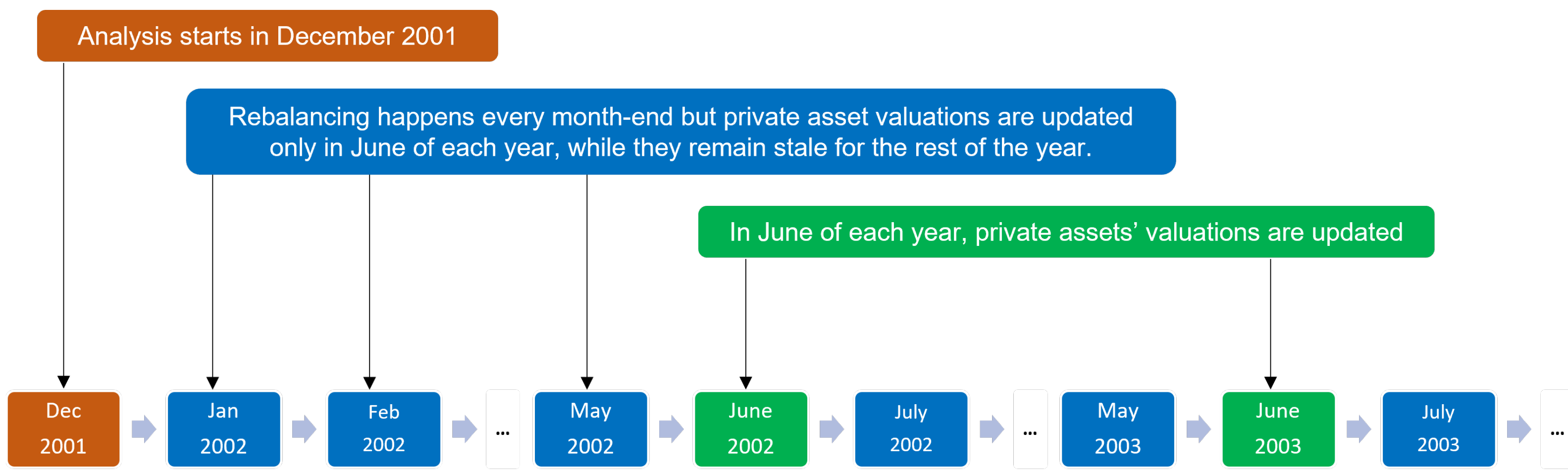
This table illustrates how the rebalancing method operates within the context of the example described in the METHODOLOGIES section. 'Observed Return' represents the observed return for the assets. 'Model Return' represents the factor model return for the private assets between official valuations. These returns are applied to the assets' NAV to compute the 'NAV - before rebalance' amounts. 'w - before rebalance' is calculated by dividing 'NAV - before rebalance' by the Total NAV before rebalance. 'NAV - after rebalance' is determined by adding the rebalance amounts to 'NAV - before rebalance'. Δw is the difference between the weight after rebalancing and the initial target weight for a given asset. At time 1, since the portfolio NAVs differ when using the observed and model returns, the portfolio weights based on the actual NAV are also provided for reference purposes. At time 2, the observed returns for private assets are applied to the time 0 observed asset values. After the values of the private assets are updated, the Model NAV is the same as the Actual NAV.

Exhibit 6: Factor Exposures

	Using Actual NAV			Using Model NAV		
	Equity Factor	**Real Rate Factor**	**Inflation Factor**	**Equity Factor**	**Real Rate Factor**	**Inflation Factor**
Time 0						
Target Exposures	110.4%	36.9%	-22.0%	- same as using Actual NAV-		
Time 1 - before rebalance						
Traditional	100.3%	40.3%	-24.6%	- Not Applicable -		
Beta	100.3%	40.3%	-24.6%	98.4%	42.2%	-25.1%
Factor	100.3%	40.3%	-24.6%	98.4%	42.2%	-25.1%
Time 1 - after rebalance						
Traditional	121.4%	36.8%	-23.2%	- Not Applicable -		
Beta	118.0%	35.2%	-22.4%	117.0%	36.9%	-22.8%
Factor	111.7%	35.2%	-21.6%	110.4%	36.9%	-22.0%
Time 2 - before rebalance						
Traditional	120.6%	38.2%	-23.5%	- Not Applicable -		
Beta	117.2%	36.5%	-22.7%	- same as using Actual NAV-		
Factor	110.7%	36.5%	-21.9%	- same as using Actual NAV-		
Time 2 - after rebalance						
Traditional	117.9%	36.9%	-22.9%	- Not Applicable -		
Beta	117.9%	36.9%	-22.9%	- same as using Actual NAV-		
Factor	110.4%	36.9%	-22.0%	- same as using Actual NAV-		

This table shows the factor exposures of the three rebalancing strategies at time 0, 1 and 2.

Exhibit 7: Historical analysis: a visualization



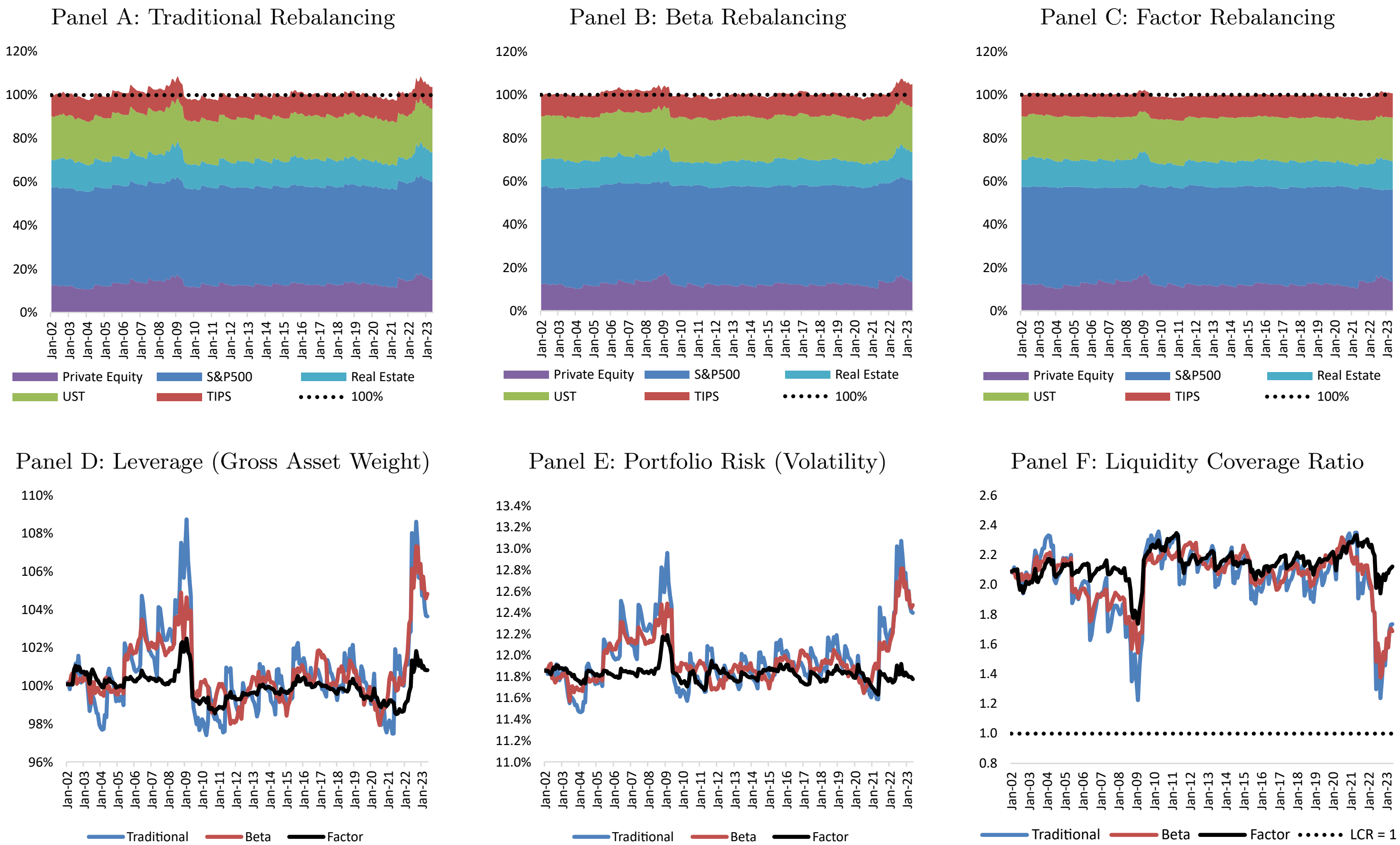
This exhibits illustrates the frequency of portfolio rebalancing, which occurs monthly, alongside the annual update schedule for private assets' valuations, specifically in June each year.

Exhibit 8: Turnover for public asset classes

	Traditional	**Beta**	**Factor**
S&P500	1.32%	1.01%	1.16%
TIPS	0.31%	0.33%	0.38%
UST	0.75%	0.83%	0.79%
Cash	0.97%	0.53%	0.24%

This table shows the turnover for the public asset classes. Turnover is defined as the standard deviation of the rebalancing amounts expressed as a percentage of NAV. The Target portfolio has the following weights: 45% in S&P 500, 10% in TIPS, 20% in U.S. Treasuries, 12.5% in Private Equity and 12.5% in Real Estate.

Exhibit 9: Asset Weights, Leverage, Risk and LCR over time



Panels A, B, and C illustrate the asset weights in our historical simulation using three different rebalancing strategies. The Target portfolio is composed of the following weights: 45% in the S&P 500, 10% in TIPS, 20% in U.S. Treasuries, 12.5% in Private Equity, and 12.5% in Real Estate. Panel D displays the leverage of the portfolio -measured as gross asset weight— across these three rebalancing strategies. Panels E and F, utilizing the same three rebalancing strategies, show the portfolio risk (volatility) calculated using the Capital Market Assumptions and the portfolio's Liquidity Coverage Ratio (LCR), respectively.

Exhibit 10: Portfolio Weights' and Risk metrics - Distribution

	Traditional			**Beta**			**Factor**		
Percentiles of metrics	*5%*	*50%*	*95%*	*5%*	*50%*	*95%*	*5%*	*50%*	*95%*
Panel A: Portfolio weights' metrics									
Risk Assets									
Private Equity	11.3%	12.8%	16.0%	10.8%	12.3%	15.3%	10.8%	12.3%	15.3%
S&P500	45.0%	45.0%	45.0%	44.6%	45.5%	46.9%	41.7%	45.2%	46.3%
Real Estate	10.9%	12.3%	14.3%	10.6%	12.1%	14.1%	10.6%	12.1%	14.1%
Risk Assets sub-portfolio	67.8%	70.3%	75.5%	68.3%	69.8%	74.5%	67.7%	69.5%	71.2%
Bonds									
UST	20.0%	20.0%	20.0%	19.8%	20.2%	20.8%	19.5%	20.2%	20.7%
TIPS	10.0%	10.0%	10.0%	9.9%	10.1%	10.4%	9.8%	10.3%	11.0%
Bonds sub-portfolio	30.0%	30.0%	30.0%	29.8%	30.4%	31.3%	29.6%	30.4%	31.4%
Cash	-5.7%	-0.4%	2.1%	-4.8%	-0.3%	1.4%	-1.2%	0.0%	1.2%
Panel B: Risk metrics									
Portfolio Volatility	11.6%	11.9%	12.6%	11.7%	11.9%	12.5%	11.7%	11.8%	11.9%
LCR	1.53	2.06	2.31	1.66	2.10	2.26	1.97	2.14	2.29

This table displays various metrics related to portfolio weights and associated risks, such as volatility and the Liquidity Coverage Ratio (LCR). Panel A presents the 5th, 50th (median), and 95th percentiles of portfolio weights across the historical period, categorized into Risk Assets (S&P 500, Private Equity, and Real Estate), Bonds (UST and TIPS), and Cash. Panel B shows two risk metrics: portfolio volatility and the Liquidity Coverage Ratio (LCR).

Exhibit 11: Portfolio Weights' and Risk metrics - Crisis Scenarios

	Traditional Rebalancing				Beta Rebalancing				Factor Rebalancing			
	2008 GFC		*Crash of 2022*		*2008 GFC*		*Crash of 2022*		*2008 GFC*		*Crash of 2022*	
	From:	*To:*	*From:*	*To:*	*From:*	*To:*	*From:*	*To:*	*From:*	*To:*	*From:*	*To:*
Crisis Scenario	May-08	Feb-09	Nov-21	Sep-22	May-08	Feb-09	Nov-21	Sep-22	May-08	Feb-09	Nov-21	Sep-22
Panel A: Portfolio weights' metrics												
Risk Assets												
Private Equity	14.1%	17.5%	14.6%	18.2%	13.4%	17.7%	13.2%	16.6%	13.4%	17.5%	13.2%	16.5%
S&P500	45.0%	45.0%	45.0%	45.0%	45.6%	42.5%	45.7%	45.3%	43.5%	40.8%	44.0%	39.8%
Real Estate	13.0%	16.3%	10.9%	15.4%	12.8%	16.0%	10.7%	15.2%	12.8%	15.8%	10.7%	15.2%
Risk Assets Subtotal	72.2%	78.8%	70.4%	78.6%	71.9%	76.1%	69.6%	77.2%	69.7%	74.2%	67.9%	71.5%
Bonds												
UST	20.0%	20.0%	20.0%	20.0%	20.3%	18.9%	20.3%	20.1%	19.9%	18.4%	20.2%	19.2%
TIPS	10.0%	10.0%	10.0%	10.0%	10.1%	9.4%	10.2%	10.1%	10.6%	9.8%	10.5%	11.2%
Bonds Subtotal	30.0%	30.0%	30.0%	30.0%	30.4%	28.3%	30.5%	30.2%	30.5%	28.2%	30.8%	30.4%
Cash	-2.2%	-8.8%	-0.4%	-8.6%	-2.3%	-4.4%	-0.0%	-7.4%	-0.3%	-2.3%	1.4%	-1.9%
Panel B: Risk metrics												
Portfolio Volatility	12.2%	13.0%	12.2%	13.1%	12.2%	12.5%	12.0%	12.8%	11.8%	12.2%	11.8%	11.9%
LCR	1.88	1.22	2.06	1.24	1.90	1.54	2.13	1.38	2.11	1.74	2.29	1.94

This table displays various metrics related to portfolio weights and associated risks, such as volatility and the Liquidity Coverage Ratio (LCR). Panel A categorizes portfolio weights into Risk Assets (S&P 500, Private Equity, and Real Estate), Bonds (UST and TIPS), and Cash, comparing these categories at the start and end of the crisis periods. Panel B highlights two risk metrics: portfolio volatility and the Liquidity Coverage Ratio (LCR).