Evaluating the Efficiency of 'Smart Beta' Indexes

Michael Hunstad, Jordan Dekhayser[1]

This draft: October 14, 2014

Abstract

The past several years has witnessed the introduction of hundreds of so-called 'smart beta' equity indexes. These indexes provide exposure to risk factors, such as value or low volatility, in order to seek excess return and/or risk reduction compared to cap-weighted indexes. Although the set of risk factors that these indexes target is relatively small, construction methodologies and historical performance have varied significantly, even among those indexes seeking exposure to exactly the same factors. In this paper we introduce a simple metric we call the Factor Efficiency Ratio that gauges the amount of active risk an index product derives from intentional, desired factor exposure versus active risk stemming from unintended, undesired bets. This ratio is a measure of how efficiently an index targets a group of intended factors and can be used to compare 'smart beta' indexes within a peer group. In doing so, we highlight several potential problems with the design of existing 'smart beta' indexes.

[1] Michael Hunstad (mrh12@ntrs.com) is the Head of Quantitative Research at Northern Trust Asset Management. Jordan Dekhayser (jdd3@ntrs.com) is a Quantitative Research Analyst at Northern Trust Asset Management.

Introduction

So-called 'smart beta' indexes that provide exposure to specific risk factors have gained significant traction recently.[1] These approaches focus on factors where substantial empirical evidence exists suggesting that they may provide positive risk-adjusted returns that modern portfolio theory, which is the theoretical basis for the cap-weighted index, does not explain. There is no definitive list of this sub-set of factors, but a survey of new index and product launches[2] indicates the following list has piqued investor interest: size, value, volatility, momentum, dividend yield, and quality.

Yet reading through the various index methodologies shows substantial differences in construction techniques, even when indexes target the same factor. The effects of these different approaches can be difficult to untangle, even if the mathematical properties are well understood. Previous work comparing equity index approaches focused on construction and ex-post risk and return characteristics.[3] This information is, of course, crucial in the investment decision making process, but does not provide a useful ex-ante cost-benefit analysis.

When an index moves away from cap-weighting to attain factor exposure, there are necessarily undesired exposures that come along for the ride. For example, when the volatility factor is targeted, certain industries (such as those in the utilities sector) tend to see an increase in weighting. Indexes that do not manage this trade-off well may or may not provide positive risk-adjusted returns to a cap-weighted index, but the results may be due to these other unintended and potentially large bets. Stated another way, if an index derives a relatively large portion of its active risk from unintended exposures, then the historical performance of the index may largely be an artifact of the unintended exposures and not of the desired factors. This represents a very significant risk to investors.

In this paper, we focus on the active risk of 'smart beta' indexes compared to their parent, cap-weighted indexes in order to analyze how well each index manages the desired-undesired (intended-unintended) tradeoff. Specifically, we compute a Factor Efficiency Ratio (FER) that measures the percent of active risk coming from desired versus undesired factor exposure. For example, if an index is value oriented, how much active risk is coming from the value factor as opposed to all other factors that are in the risk model? When comparing multiple value indexes, this metric provides an unambiguous interpretation of how efficient an index is at acquiring exposure to a given factor.

We will then show how indexes with higher FERs can be combined with a passive investment in a cap-weighted index to provide a more optimal result (more active risk coming from the desired factor). And finally, we discuss what the FER can tell us about performance. The majority of indexes we looked at have only been around for a short time, so we conduct a hypothetical exercise in order to see what returns may have looked like if the point-in-time FER we present in this paper were held historically.

The Factor Efficiency Ratio

The ex-ante active risk of a portfolio, or tracking error, measures a portfolio's expected variation in performance relative to a benchmark. This active risk is generally estimated with a risk model that focuses on several common sources of risk, or factors, as well as estimates for stock specific risks that the common factors do not explain. The Factor Efficiency Ratio (FER) measures the ratio of active risk coming from desired, intended factor exposures to undesired, unintended factor exposures, and is calculated as

$$FER = \frac{\sum FC_d}{AR_t - \sum FC_d} \tag{1}$$

where the *FER* is the Factor Efficiency Ratio, FC_d is the active risk contribution of d desired factors, and AR_t is the total active risk of the portfolio to the benchmark.

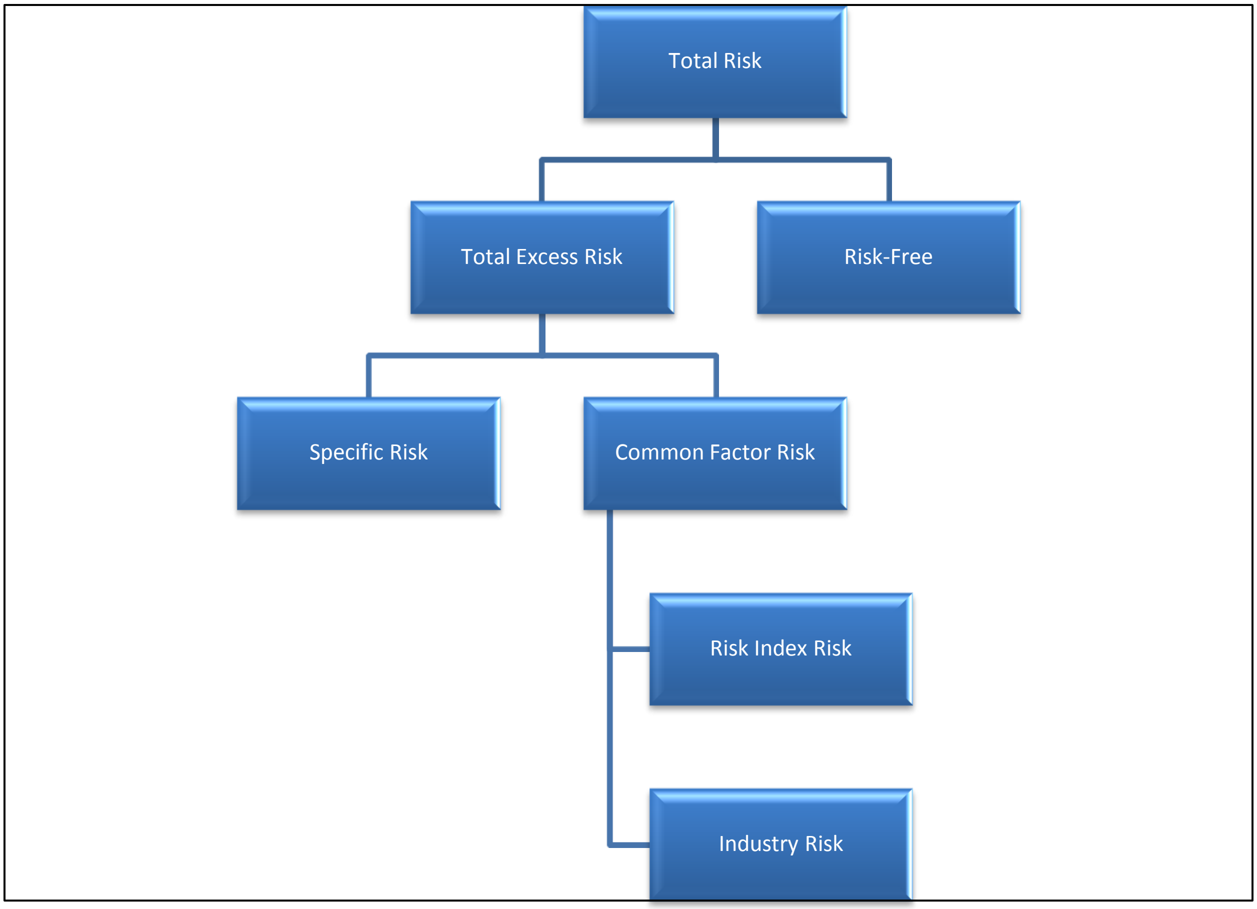
The total active risk of a portfolio equals the sum of the individual active risk contributions from each common factor plus the sum of the stock specific active risk contributions. To calculate the active risk contribution of a common factor or a specific stock, we utilize the x-sigma-rho approach which shows that active risk contribution for a factor is the product of the active factor exposures of the portfolio (x) times the factor volatilities (sigma) times the correlation of the factor's returns to the active portfolio (rho).[4] The total active risk for a portfolio (AR_t) can thus be represented using the following x-sigma-rho formulation

$$\sigma(AR_t) = \sum_i X_i^A \sigma(f_i)\rho(f_i, AR_t) + \sum_k w_k^a \sigma(u_k)\rho(u_k, AR_t) \tag{2}$$

where $\sigma(AR_t)$ is the active risk of portfolio t, X_i^A is the active exposure of the portfolio to the i-th factor, $\sigma(f_i)$ is the volatility of factor i's returns, $\rho(f_i, AR_t)$ is the correlation of the i-th factor to the active return of the portfolio, w_k^a is the active weight of stock k, $\sigma(u_k)$ is the volatility of stock k's specific returns, and (u_k, AR_t) is the correlation of the specific returns of stock k to the active return of the portfolio.

The first part of the right hand side of equation (2) represents all 'common' factors. These common factors are then decomposed, or grouped, into similar risks. For example, we display a risk decomposition tree for a risk model that was used in our analysis (Barra's USE3). Total risk in USE3 is broken down into the excess risk and the risk-free risk. Excess risk is decomposed into two main categories: common factor risk and specific risk - second part of equation (2). Common factor risk is further decomposed into risk index risk (also known as style factors) and industry risk. USE3 has 52 industry factors and 13 risk index factors. The desired factors that are used in 'smart beta' indexes will typically live in the risk index bucket (such as value, volatility, dividend yield). For our analysis, we are concerned with active, as opposed to total, risk, but the hierarchy is similar. See **Figure 1**, which is reproduced from Figure 4-2 in the USE3 risk model handbook.

Figure 1: Total risk decomposition (USE3)



Data

Multiple sources were used for index holdings information. Index constituents from the index provider were preferred. If these were unavailable to us, we used portfolio holdings of an ETF that passively tracks a given 'smart beta' index. All data is as of 31 December 2013.

The benchmark chosen to calculate the FER for each product was based on the universe used by that product. For example, we measure the active risk of Russell 1000 Value against Russell 1000 and MSCI USA Value against MSCI USA. For a risk model, we utilize Barra's USE3 (GEM2) was used for domestic (international) indexes. The choice of benchmark and risk model will impact results, and we would recommend that investors use ones that align best with their investment process. However, using a benchmark other than that underlying the 'smart beta' index would likely degrade the FER (for example, we would expect the Russell 1000 High Efficiency Value index to be more efficient when measured against the Russell 1000 then, say, the S&P 500 or MSCI USA).

Three factors (value, volatility, and dividend yield) are chosen for analysis. For each factor, a number of indexes are chosen. The results are further divided into domestic and international strategies in order to capture differences in risk created by geography. The choice of factor and index is done for illustrative purposes, though we attempt to choose indexes or products that are well known or have other interesting reasons for inclusion. Inclusion or exclusion is not an endorsement of any particular approach or index and we acknowledge the indexes selected may claim other benefits outside of targeting a desired factor defined in a specific manner.

In addition to the total active risk and the FER, we also display the active factor exposures of each index as well as the $FC_{\%}$ which represents the active risk derived from the desired factor(s). A summary of our results is presented in **Table 1**.

Table 1: Summary of Factor Efficiency Ratio analysis

Index	Total Active Risk	Factor Exposure	$FC_{\%}$	FER	
Value (Domestic)					
Russell 1000 Value	2.31%	0.55	0.40%	0.21	
Russell 1000 High Efficiency Value	2.05%	0.47	0.34%	0.20	
FTSE RAFI US 1000*	1.73%	0.33	0.25%	0.17	
MSCI USA Value Weighted	1.59%	0.32	0.23%	0.17	
MSCI USA Value	2.13%	0.38	0.19%	0.10	
Value (International)					
MSCI World ex-US Value	1.29%	0.44	0.30%	0.30	
MSCI World ex-US Value Weighted	1.47%	0.34	0.21%	0.16	
Russell Developed ex-US LC High Efficiency Value	1.78%	0.47	0.17%	0.10	
Volatility (Domestic)					
Russell 1000 High Efficiency Low Volatility	3.03%	-0.39	1.60%	1.12	
MSCI USA Minimum Volatility	4.19%	-0.46	1.76%	0.72	
Russell 1000 Low Volatility*	3.83%	-0.41	1.40%	0.57	
S&P Low Volatility*	4.73%	-0.45	1.48%	0.46	
MSCI USA Risk Weighted	2.22%	-0.08	0.12%	0.06	
Volatility (International)					
Russell Developed ex-US LC High Efficiency Low Volatility	3.97%	-0.71	3.07%	3.39	
MSCI World ex-US Minimum Volatility	6.94%	-0.90	4.28%	1.60	
MSCI World ex-US Risk Weighted	3.25%	-0.30	1.14%	0.54	
					Dividend Yield%
Dividend Yield (Domestic)					
Dow Jones US Dividend Select*	4.60%	0.89	0.63%	0.16	3.63%
MSCI USA High Dividend Yield	3.55%	0.59	0.38%	0.12	3.07%
Dow Jones Dividend 100*	3.72%	0.48	0.25%	0.07	2.77%
S&P 1500 Dividend Aristocrats*	3.22%	0.39	0.19%	0.06	2.66%
NASDAQ Dividend Achievers*	3.33%	0.10	0.02%	0.01	2.07%

* - Denotes ETF holdings were used

Results

Value

Of the eight indexes representing value, the MSCI World ex-US Value index stands out as the most efficient with a FER of 0.30. A FER of 0.30 means that an investor in this index receives only 0.30% of active risk from the value factor for every 1.00% of active risk from the other undesired, unintended factors. Of course, this FER appears strikingly low! How could the *most* efficient index of the value cohort attain less than one fourth (0.30% from value factor/1.29% total active risk) of its total active risk from value exposure? **Figure 2** shows the FER as well as the active factor exposures of the selected strategies (strategies in green are international). We show the factor exposures themselves because having strong factor exposure can be mistakenly interpreted as providing efficient exposure to that factor. The factor exposures are merely the benefit side of the cost-benefit analysis.

As we will see, low FERs are typical for most factors and highlight a pervasive problem with 'smart beta' indexes. Low Factor Efficiency Ratios suggest that value exposure is coupled with many unintended exposures that dominate the active risk of the index. In particular, high value stocks tend to also have a negative exposure to the momentum factor, i.e., many cheap stocks were once expensive but have experienced price depreciation. Value based indexes (and, in fact, most 'smart beta' indexes) also tend to have a small size bias, indicating some potential double-dipping into an additional well-known factor. As a result, unless otherwise controlled, many indexes with 'value' in their name could also be more accurately characterized as negative momentum or even mid-cap indexes.

Further highlighting the difference between exposure and efficiency, the MSCI USA Value index is the least efficient with a FER of 0.10, but has exposure to the value factor (0.38) that is about average for the cohort. The low FER for this index stems not from exposure to value but rather to large unintended exposures, especially idiosyncratic or specific risk as shown in **Table 2**. For this reason, the MSCI USA Value index appears relatively inefficient compared to other indexes in the cohort - but this conclusion is justified in the sense that the index takes *three times* as much active risk from specific risk (0.65%) as it does from the value factor (0.19%).

While the ability of the value factor to generate positive excess returns has been repeatedly demonstrated[5], the low efficiency of 'smart beta' indexes targeting value gives us pause. It would seem that much may be lost in the translation between theoretical value factor performance and actual 'smart beta' products.

Figure 2: Factor Efficiency Ratios for selected value indexes

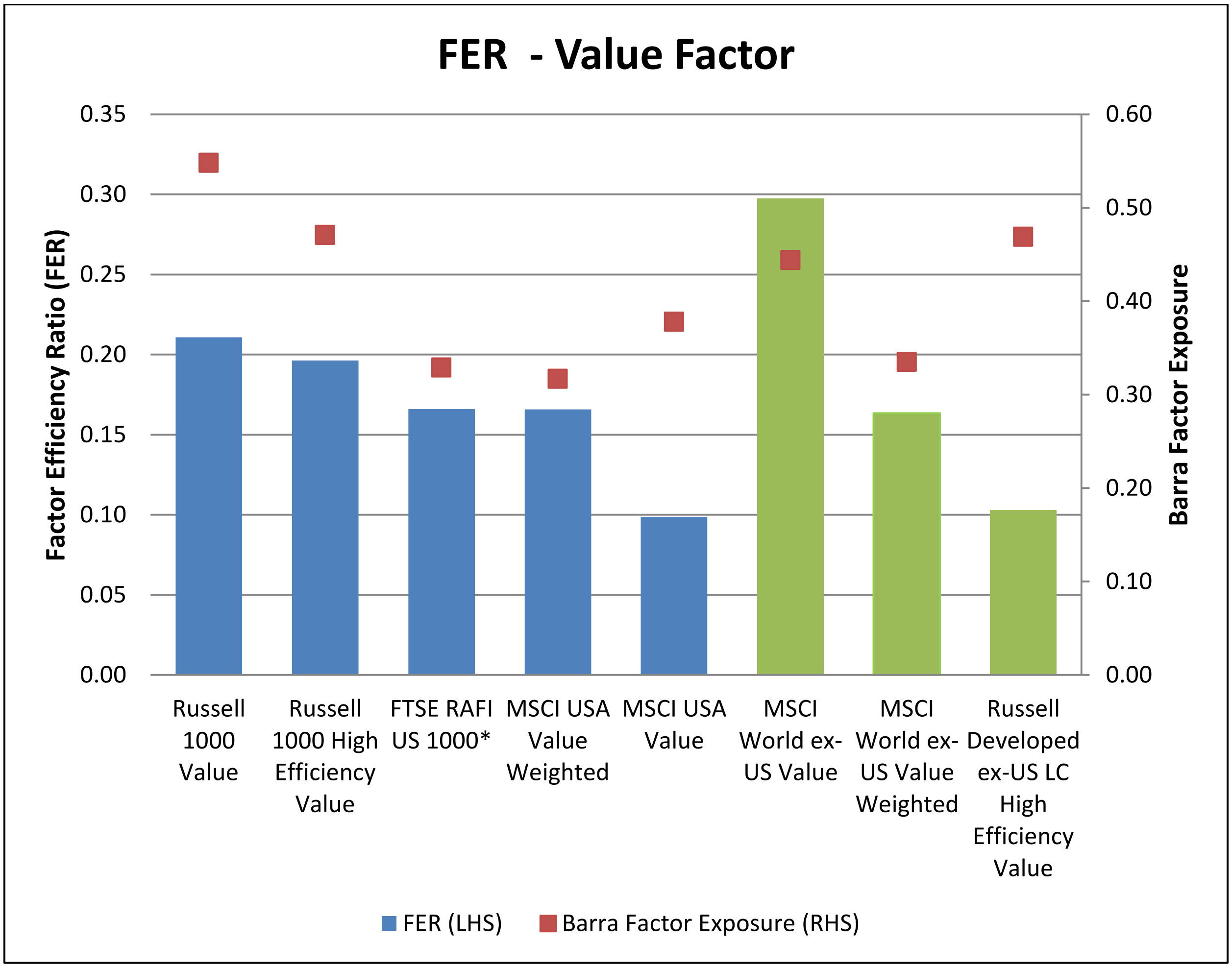


Table 2: Breakdown of Factor Efficiency Ratios for value factor

		Desired $FC_{\%}$	Common $FC_{\%}$			
Index	**FER**	**Value**	**Non Risk Indices**	**Risk Indices**	**Specific**	**AR_t**
Domestic						
Russell 1000 Value	0.21	0.40%	0.83%	1.03%	0.46%	2.31%
Russell 1000 High Efficiency Value	0.20	0.34%	0.55%	1.27%	0.23%	2.05%
FTSE RAFI US 1000	0.17	0.25%	0.45%	0.93%	0.35%	1.73%
MSCI USA Value Weighted	0.17	0.23%	0.49%	0.83%	0.27%	1.59%
MSCI USA Value	0.10	0.19%	0.57%	0.90%	0.65%	2.13%
International						
MSCI World ex-US Value	0.30	0.30%	0.30%	0.53%	0.47%	1.29%
MSCI World ex-US Value Weighted	0.16	0.21%	0.28%	1.05%	0.15%	1.47%
Russell Developed ex-US LC High Efficiency Value	0.10	0.17%	1.10%	0.44%	0.24%	1.78%

Low Volatility

Low volatility indexes tend to take more active risk, but also tend to be somewhat more efficient than other factor indexes. Of the eight low volatility indexes analyzed, active risk ranges from 2.2% to more than 6.9% with FERs ranging from just 0.06 all the way up to 3.39. A brief discussion on methodologies provides useful insights to why FERs may have such a wide range.

The low volatility strategies analyzed here can be placed into two broad categories: those that ignore stock correlations, and those that incorporate them. The former approach has its roots in risk parity which seeks an equal risk contribution from each asset; the latter approach has its roots in the minimum variance portfolio and seeks an equal marginal contribution from each asset in the portfolio to total risk, such that changing the weights of the stocks in the portfolio would necessarily increase total risk.[6]

The two approaches will produce equivalent portfolios in the special case where all off-diagonal elements of the covariance matrix are zero. To emphasize, this special case requires that *all* off-diagonals are zero, not merely the *average*. In fact, we estimate that the actual average pairwise correlation of the Russell 1000 was 0.31 as of 31 December 2013[7] which, along with intuition, indicates that it is highly unlikely that the correlation of all pairs of stocks are equal to zero.

The constituent weights of the MSCI USA Risk Weighted, S&P Low Volatility, Russell 1000 Low Volatility and High Efficiency Low Volatility, MSCI World ex-US Risk Weighted and Russell Developed ex-US LC High Efficiency Low Volatility indexes all depend on constituent level volatilities as a key input in their construction without incorporating correlation information. Their approach places them in the first category discussed in the previous paragraph. The MSCI USA Minimum Volatility and MSCI ex-US Minimum Volatility indexes use historical volatility as an input, but also incorporate stock correlations through modeling techniques that require an estimated covariance matrix, which places them in the second category.

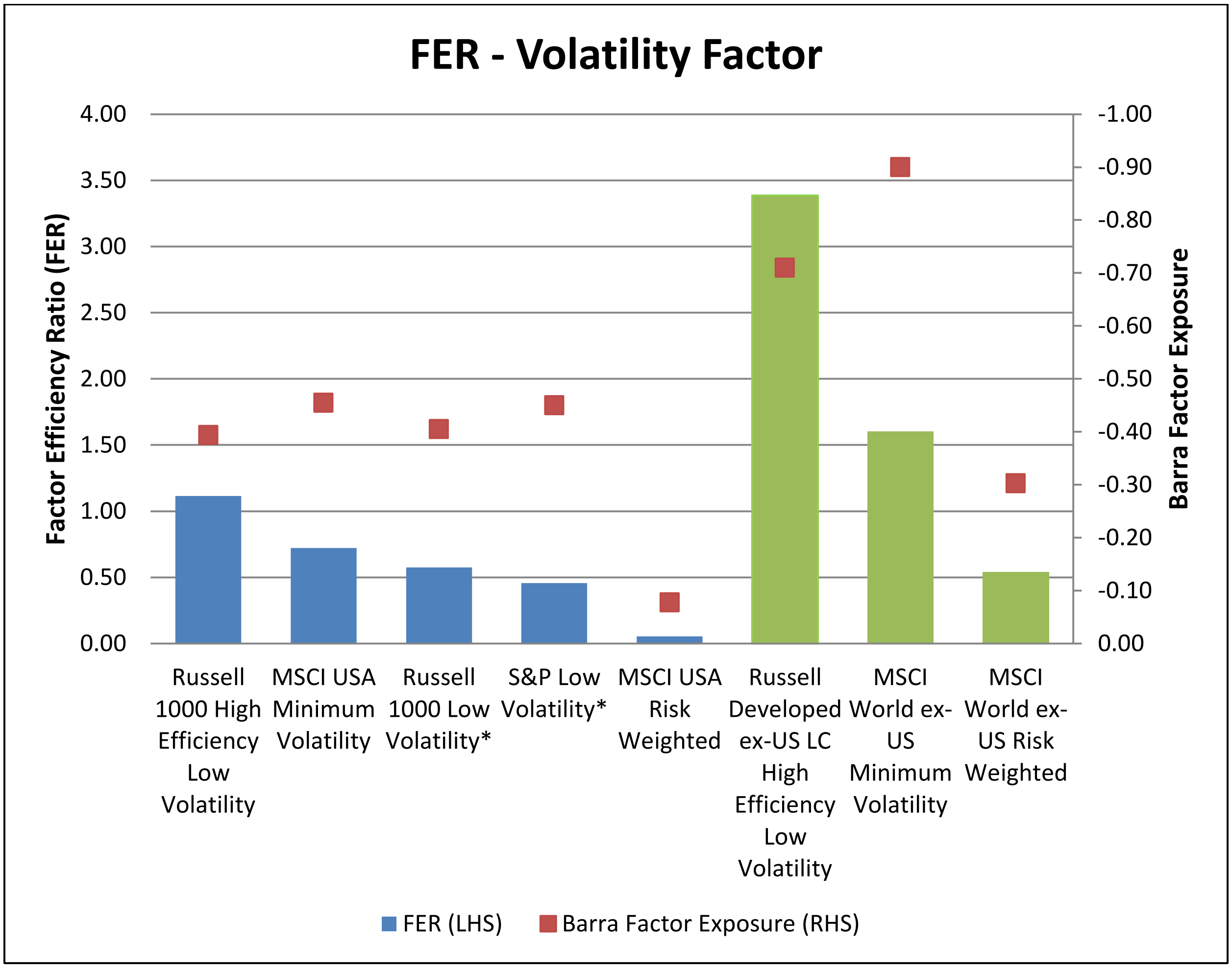
Further confusing the issue, the approaches that fall into the first category (ignore stock correlations) get to their final constituent weights through dissimilar methods, although all utilize some measure of historical volatility. The MSCI USA and ex-US Risk Weighted, Russell 1000 Low Volatility, and S&P 500 Low Volatility indexes all weight constituents on the inverse of squared volatilities. However, S&P 500 Low Volatility only includes the top 100 names (that is, the 100 names with the lowest historical volatility) and the Russell HELV indexes take a slightly different approach by turning historical volatilities into factor scores and setting breakpoints that determine whether to overweight, underweight, or exclude a particular stock.

It is unclear, ex-ante, why one approach would be superior to another on a risk-adjusted basis. However, through the FER lens, we are at least able to determine which approach derives more active risk from the volatility factor per unit of unintended active risk. **Figure 3** and **Table 3** show the FERs and factor exposures for each of the indexes and their breakdown among the different components, respectively. Table 3 groups the indexes by the two categories discussed above.

Like value indexes, some low volatility indexes are plagued by significant unintended exposures. The average FER for low volatility indexes is greater than 1, suggesting that as a group, these indexes do a modest job of managing intended and unintended bets. Yet certain low volatility indexes take substantial industry bets, which exposes investors to return profiles not related to the volatility factor. From a factor perspective, low volatility indexes tend to have small size and low liquidity biases. This can be seen in Table 3 as contributors to the large values under Risk Indexes. In addition, many low volatility indexes have significant industry and sector biases, especially to utilities. These exposures are included in the Non Risk Indexes figure shown in Table 3.

Despite growing research confirming excess returns to low volatility stocks[8], large unintended exposures exist in all low volatility indexes. As discussed above, most index construction methodologies are relatively simple and contain no mechanism to deal with unintended exposures. As a result, efficiency can vary dramatically across low volatility indexes giving investors strong reason to be concerned about where returns are coming from when they invest in a low volatility, ‘smart beta’ index.

Figure 3: Factor Efficiency Ratios for selected low volatility indexes



Note: RHS is inverted, as more negative exposure to the volatility factor is what is 'desired'

Table 3: Breakdown of Factor Efficiency Ratios for volatility factor

Index	FER	Desired $FC_{\%}$ Volatility	Common $FC_{\%}$ Non Risk Indices	Risk Indices	Specific	AR_t
Ignores correlations						
Domestic						
Russell 1000 High Efficiency Low Volatility	1.12	1.60%	0.78%	2.07%	0.18%	3.03%
Russell 1000 Low Volatility	0.57	1.40%	1.43%	1.74%	0.66%	3.83%
S&P Low Volatility	0.46	1.48%	2.09%	2.16%	0.48%	4.73%
MSCI USA Risk Weighted	0.06	0.12%	0.67%	1.06%	0.49%	2.22%
International						
Russell Developed ex-US LC High Efficiency Low Volatility	3.39	3.07%	0.58%	3.28%	0.11%	3.97%
MSCI World ex-US Risk Weighted	0.54	1.14%	1.49%	1.60%	0.17%	3.25%
Incorporates correlations						
Domestic						
MSCI USA Minimum Volatility	0.72	1.76%	1.60%	2.14%	0.46%	4.19%
International						
MSCI World ex-US Minimum Volatility	1.60	4.28%	2.22%	4.53%	0.19%	6.94%

Dividend yield

Of the factors presented in this paper, dividend yield indexes had the lowest Factor Efficiency Ratios with a range from 0.01 to 0.16. For all five high dividend yield indexes analyzed, the *vast* majority of active risk was attributable to unintended exposures – both common and specific (see **Table 4**). Indeed, the strongest factor exposure for many of the indexes was not dividend yield at all but some combination of volatility and size factors with a large dose of industry and specific risk. In other words, like value indexes, dividend yield indexes may be merely masquerading as dividend focused strategies with unintended exposures dominating active risk and, hence, outcomes.

We will use the dividend yield factor to illustrate an important point highlighted earlier in this paper related to cost-benefit analysis. Exposure to the dividend factor is the benefit side of the cost-benefit analysis. When we look at the five indexes analyzed, there does appear to be a monotonic relationship between the factor exposure and the FER. However, if we take this idea of maximizing factor exposure to the extreme, consider a portfolio that maximizes dividend yield without constraint. Just as a portfolio's expected return will be maximized through a 100% investment in the stock with the highest expected return, a portfolio's dividend yield will be maximized through a 100% investment in the stock with the highest dividend yield.

On 31 December 2013, the American Capital Agency Corp (ticker: AGNC) had a dividend yield of 13.48%, which was the highest dividend yield in the Russell 1000.[9] The FER of this portfolio is 0.07 because although this portfolio has a very high $FC_{\%}$ of 1.52% (compared to an average of 0.30% across the other 5 portfolios), it comes with a total active risk of 23.94%. Interestingly, despite obvious diversification issues, a 100% investment in the single stock portfolio is more efficient than two of the dividend yield factor based indexes under analysis (NASDAQ Dividend Achievers and S&P 1500 Dividend Aristocrats) and is similar to the Dow Jones Dividend 100 index.

To sum up, although the dividend yield indexes we analyze certainly deliver higher dividend yields for those seeking income, the cost to doing so is fairly significant in terms of undesired exposures. After all, if we can say that a 100% holding in a single stock may be as, or more, efficient than a 'smart beta' index, then perhaps investors may find that there are more efficient ways to harvesting the dividend yield factor.

Figure 4 shows FERs and factor exposures and Table 4 shows a further breakdown of the five dividend yield strategies analyzed. We have included our hypothetical single stock portfolio in Table 4 for illustrative purposes.

Figure 4: Factor Efficiency Ratios for selected dividend yield indexes

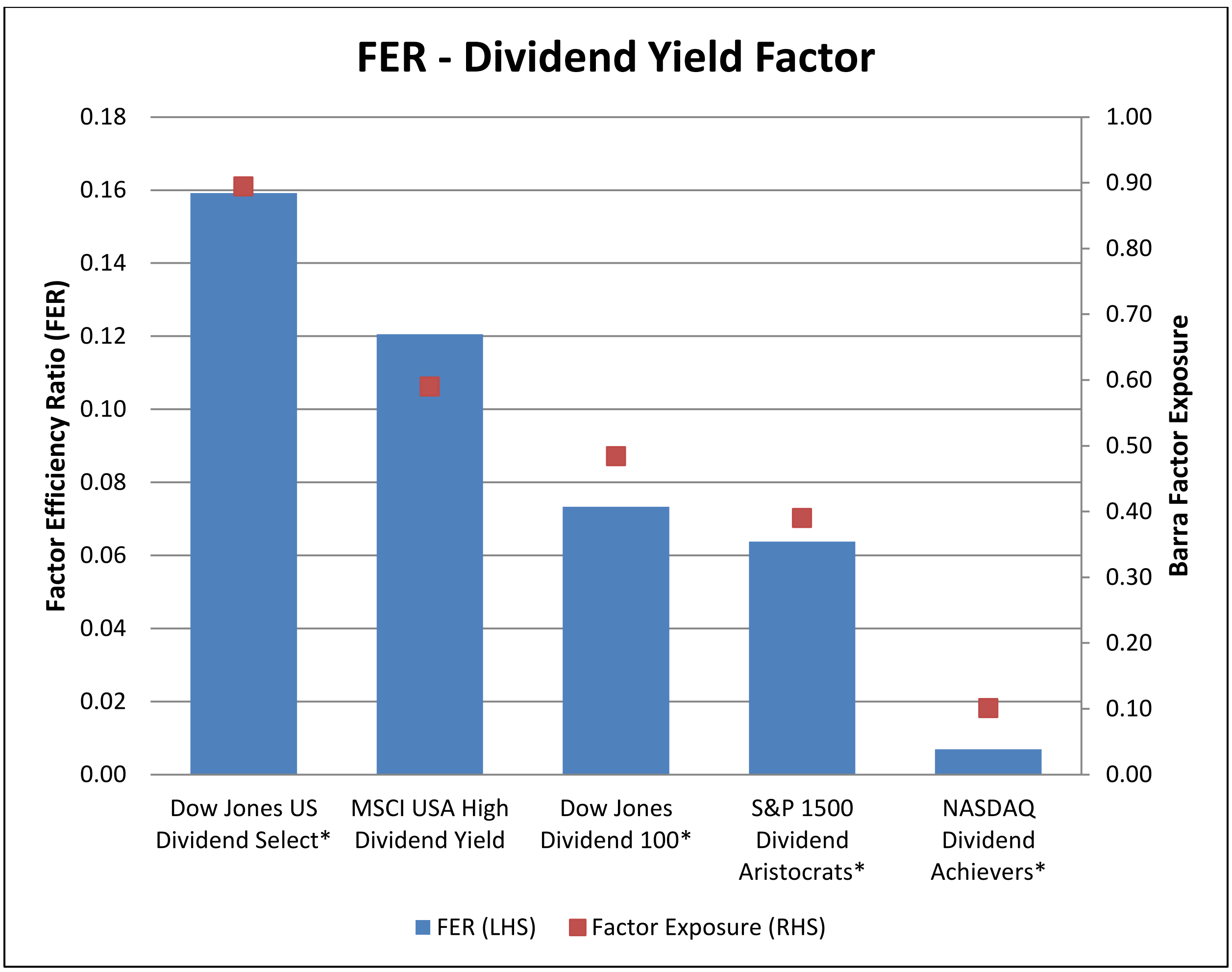


Table 4: Breakdown of Factor Efficiency Ratios for dividend yield factor

Index	FER	Desired $FC_{\%}$ Dividend Yield	Common $FC_{\%}$ Non Risk Indices	Risk Indices	Specific	AR_t	Dividend Yield %
100% allocation to highest div yld stock	**0.07**	**1.52%**	**3.49%**	**10.19%**	**10.26%**	**23.94%**	**13.48%**
Dow Jones US Dividend Select	0.16	0.63%	2.25%	1.68%	0.67%	4.60%	3.63%
MSCI USA High Dividend Yield	0.12	0.38%	1.17%	1.64%	0.74%	3.55%	3.07%
Dow Jones Dividend 100	0.07	0.25%	0.75%	2.06%	0.91%	3.72%	2.77%
S&P 1500 Dividend Aristocrats	0.06	0.19%	0.71%	1.59%	0.92%	3.22%	2.66%
NASDAQ Dividend Achievers	0.01	0.02%	0.70%	1.65%	0.98%	3.33%	2.07%

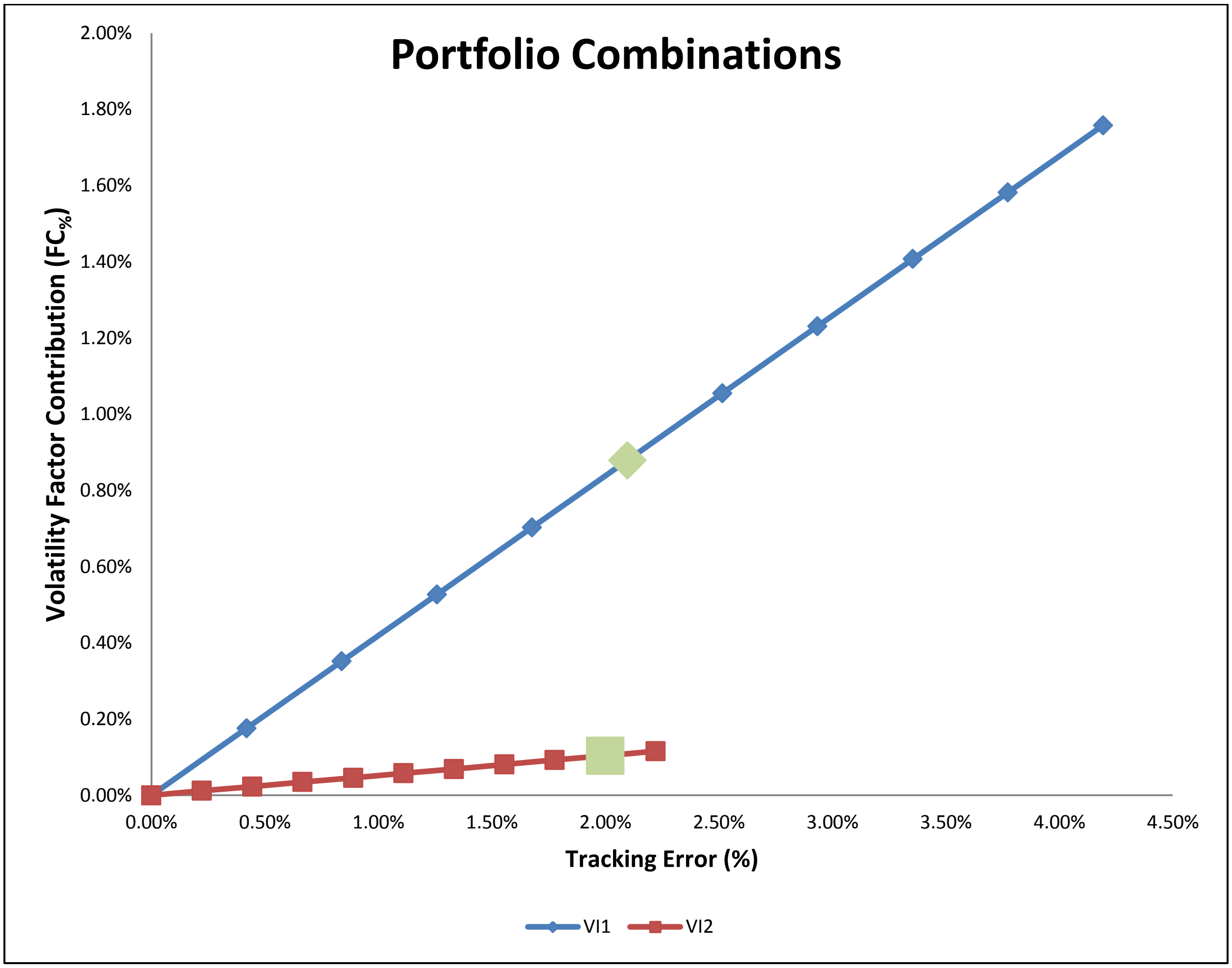
Optimizing the active risk budget for a factor based approach

In this section, we look at the tradeoffs between active risk and desired factor exposure. In **Table 5** we show the effects of combining the MSCI USA with one of two low volatility indexes – MSCI USA Minimum Volatility (VI_1) or MSCI USA Risk Weighted (VI_2). As shown, because VI_1 has a higher FER, it is more efficient in the sense that it always produces a higher active risk contribution from the volatility factor than VI_2 per unit of tracking error. For example, for an active risk of around 2.00%, the combination with VI_1 provides 0.88% of active risk from the volatility factor versus only 0.10% when VI_2 is used. These results hold generally. **Figure 5** shows the possible combinations of $FC_{\%}$ and active risk graphically.

Table 5: Efficient combination of portfolios

		VI_1		VI_2	
MSCI USA	*Volatility Index*	*$FC_{\%}$*	*Active Risk*	*$FC_{\%}$*	*Active Risk*
100%	0%	0.00%	0.00%	0.00%	0.00%
90%	10%	0.18%	0.42%	0.01%	0.22%
80%	20%	0.35%	0.84%	0.02%	0.44%
70%	30%	0.53%	1.26%	0.04%	0.67%
60%	40%	0.70%	1.68%	0.05%	0.89%
50%	50%	0.88%	2.10%	0.06%	1.11%
40%	60%	1.06%	2.52%	0.07%	1.33%
30%	70%	1.23%	2.94%	0.08%	1.56%
20%	80%	1.41%	3.36%	0.09%	1.78%
10%	90%	1.58%	3.77%	0.10%	2.00%
0%	100%	1.76%	4.19%	0.12%	2.22%
		VI_1 FER =>	0.72	VI_2 FER =>	0.06

Figure 5: Portfolio combinations



Note: Each dot represents a 10% increase in allocation to the volatility strategy (VI1 or VI2). The origin represents a zero percent allocation to the volatility strategy.

The two main conclusions to draw from this analysis are: (1) higher FER portfolios achieve more efficient factor exposure, all else equal, when they are combined with a portfolio that is neutral to factor exposures (the cap-weighted index is neutral in the sense that all active exposures are zero) and (2) the optimal portfolio through the FER lens is one that provides the maximum $FC_{\%}$ per unit of tracking error. This optimal portfolio delivers the maximum active risk from the desired factor while accounting for other inherent risks to the portfolio.

Performance

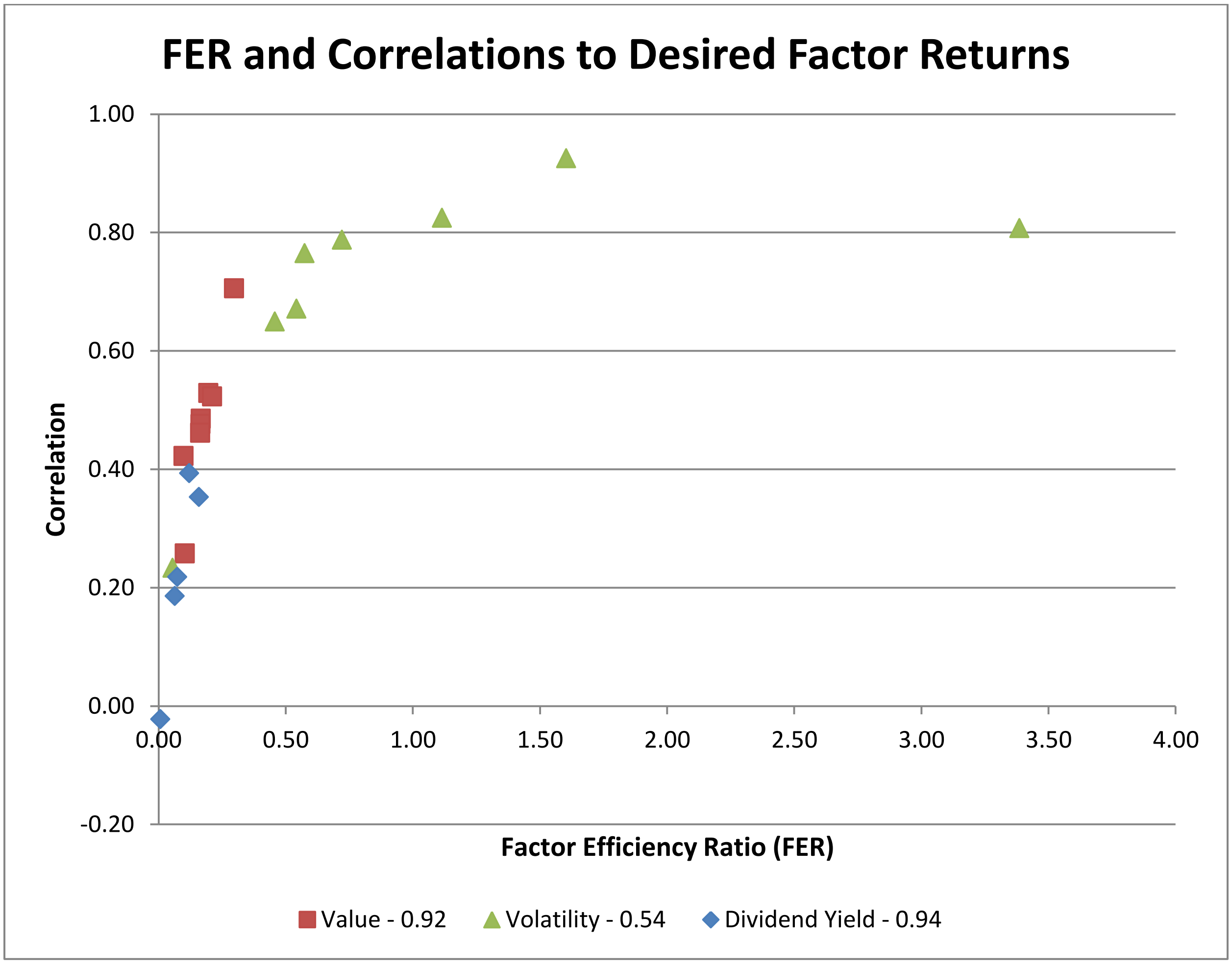
'Smart beta' indexes with higher FERs should generate higher risk-adjusted excess returns. Recall that the numerator of the FER is the active risk contribution of the desired or intended factor(s). The denominator of the FER is the active risk contribution from unintended factors. If extraneous unintended exposures are reduced then total active risk should also be reduced (FER will increase). Indexes with high FERs have high intended exposures and/or low unintended which should, in principle, translate to higher risk adjusted returns, given many of these unintended exposures have zero or even negative expected returns.

Unfortunately, most ‘smart beta’ indexes have relatively short track records thereby reducing the usefulness of a traditional risk and return analysis. To address how the FERs relate to returns, we compute the returns of hypothetical portfolios using the active factor exposures as of 31 December 2013, and assume these exposures have been constant through time. Next, we calculate the correlation of these portfolios to the factor return series for each factor being analyzed (value, low volatility, dividend yield) over a period covering multiple business cycles (from December 1973 and 1997 for domestic and international indexes, respectively, through May 2014). Finally, we plot these correlations against our ex-ante measure of factor efficiency, the FER.

These results are shown in **Figure 6**.[10] Note that this approach looks at the *active* factor exposures and is net of the benchmark returns. Thus, we are looking at the *incremental* returns that would result from the active factor exposures of a strategy. In an ideal world, the active return of a portfolio would have a 1 correlation to the desired factor, indicating that all of the active return was coming from the desired factor exposure. In practice, other factor exposures impact returns and move this correlation away from 1.

As shown in the exhibit, there is a strong positive relationship between our ex-ante measure of factor efficiency and how well an index captures intended factor returns. The correlations from the regression are 0.94, 0.92, and 0.54 for dividend yield, value, and volatility, respectively. Because each factor has its own range, we show the correlations when looking at each factor as its own sub-set.

Figure 6: Factor Efficiency Ratio and correlations to desired factor returns



Note: Analysis for domestic strategies uses USE3 factor returns from December 1973 through May 2014; analysis for international strategies uses GEM2 factor returns from December 1997 through May 2014.

Admittedly, this form of analysis is somewhat unsatisfying as it lacks real historical index data. If data were available it would be better to regress FERs on actual risk adjusted returns or information ratios. However, our analysis does make an important point – the factor efficiency ratio (FER) is a viable measure of the 'purity' of an index's factor exposures.

The importance of index purity should not be underestimated. Anecdotally, we know many asset owners evaluating 'smart beta' strategies are doing so from an asset allocation context whereby factor returns enter their allocation models in a manner similar to any other asset class. The model recommendation is then implemented using one or more index products that can deliver the required factor exposures.

If the chosen index products are not 'pure' then their actual performance could vary markedly from that of the desired factors. As a result, the total portfolio performance may be materially different from what the asset allocation model would predict. This is a source of significant risk for asset owners and highlights the importance of index 'purity'.

Conclusions

In this paper we developed a simple metric called the Factor Efficiency Ratio (FER) to gauge how efficiently so-called ‘smart beta’ products gain exposure to desired, intended factors and avoid undesired, unintended exposures. The resulting metric allows potential investors to evaluate the relative merits of ‘smart beta’ strategies through a lens that aligns with their investment process. Investors need only define the reference benchmark and specify the risk model to be used.

In applying the Factor Efficiency Ratio to existing ‘smart beta’ indexes several important insights were gained. First, we showed that over most ‘smart beta’ indexes analyzed, Factor Efficiency Ratios were *strikingly* low. In other words, existing ‘smart beta’ indexes were generally unable to provide desired factor exposures without taking on substantial unintended exposures. We feel this problem stems from the relative simplicity of ‘smart beta’ index construction. For most indexes there exists no mechanism to control unintended bets which results in very low Factor Efficiency Ratios. These low FERs indicate these indexes are not ‘pure’ in their delivery of intended factor exposures and pose a significant risk to investors.

Second, in analyzing ‘smart beta’ indexes from a FER perspective we showed that some ‘smart beta’ indexes advertised as delivering a certain intended factor exposure, such as value, had risk profiles that were dominated by unintended exposures, including other factors such as size and volatility. In this sense we feel that many ‘smart beta’ products are in fact misrepresented or, at least, mislabeled (for example, the iShares MSCI USA Size Factor ETF – ticker SIZE - is benchmarked to the MSCI USA Risk Weighted Index). It is important that investors understand the true risk profile of indexes they use to invest and not put too much credence on index or product names and descriptions. *Caveat emptor*.

In the final sections of this paper we showed how to use the Factor Efficiency Ratio (FER) to optimize one’s tracking error budget. We also tied in a measure of ex-post factor efficacy with our ex-ante metric to demonstrate that strategies with high FERs tend to provide returns that correlate more highly with the desired factor returns and are, thus, more ‘pure’. Our analysis focused on so-called ‘smart beta’ products because their methodologies allow them to be objectively replicated, though the approach could be applied to any portfolio. We conclude that ‘smart beta’ indexes that have higher and more stable Factor Efficiency Ratios are better suited for factor investing purposes, all else equal, and that the FER can be utilized in conjunction with other analytical tools to provide insights that may not be gleaned from these tools independently.

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[1] According to two articles from the *Economist* (6 July 2013 "The rise of smart beta" and 3 May 2014 "Will invest for food"), assets invested in "smart beta" funds increased from $142 billion to $330 billion.
[2] For example, MSCI formally detailed their 'Family of Factor Indexes' in Exhibit 10 in Bender, Briand, Melas, Subramanian (2013) "Foundations of Factor Investing" which can be downloaded from http://www.msci.com/resources/pdfs/Foundations_of_Factor_Investing.pdf. Russell launched their Russell High Efficiency™ Factor Index Series in Barber, Bennett, Paris (2014), which can be downloaded from http://www.russell.com/documents/indexes/research/russell-high-efficiency-factor-index-series.pdf.
[3] Chow, Hsu, Kalesnik, and Little (2010) analyze a number of non-cap weighted approaches and conclude that, regardless of the approach, the relative performance was due to small size and high value exposure that can be achieved through naïve equal weighting.
[4] See Davis and Menchero (2010) for a formalized explanation of x-sigma-rho. Equation 2 in this paper corresponds to equation 16 in the cited paper. Note that for the stock specific elements, x is a vector of active security weights, sigma is a vector of the specific volatility, and rho is a matrix where the diagonal represents the correlations between specific returns and the active portfolio.
[5] See, for instance, Fama, French (1992).
[6] Clarke, De Silva, and Thorley (2013) explore the math, assumptions, and results of risk-based strategies. They show a variety of approaches empirically. In their Exhibit 1, Risk Parity and Minimum Variance solutions produce similar Sharpe ratios, but the Minimum Variance approach does lower risk by about 25% (12.4% versus 16.6%).
[7] Average pairwise correlation estimated as of 31 December 2013 using Axioma United States Medium Horizon Fundamental Factor Risk Model (US2AxiomaMH).
[8] See, for instance, Blitz, van Vliet (2007).
[9] Dividend yield % and the dividend yield factor are not precisely the same thing. The former reflects trailing 12 month dividends divided by the current price, while exposures to the dividend yield factors are normalized and winsorized versions of the percentage. See http://www.alacra.com/alacra/help/barra_handbook_US.pdf for details on USE3.
[10] Details are available upon request with permission from MSCI.