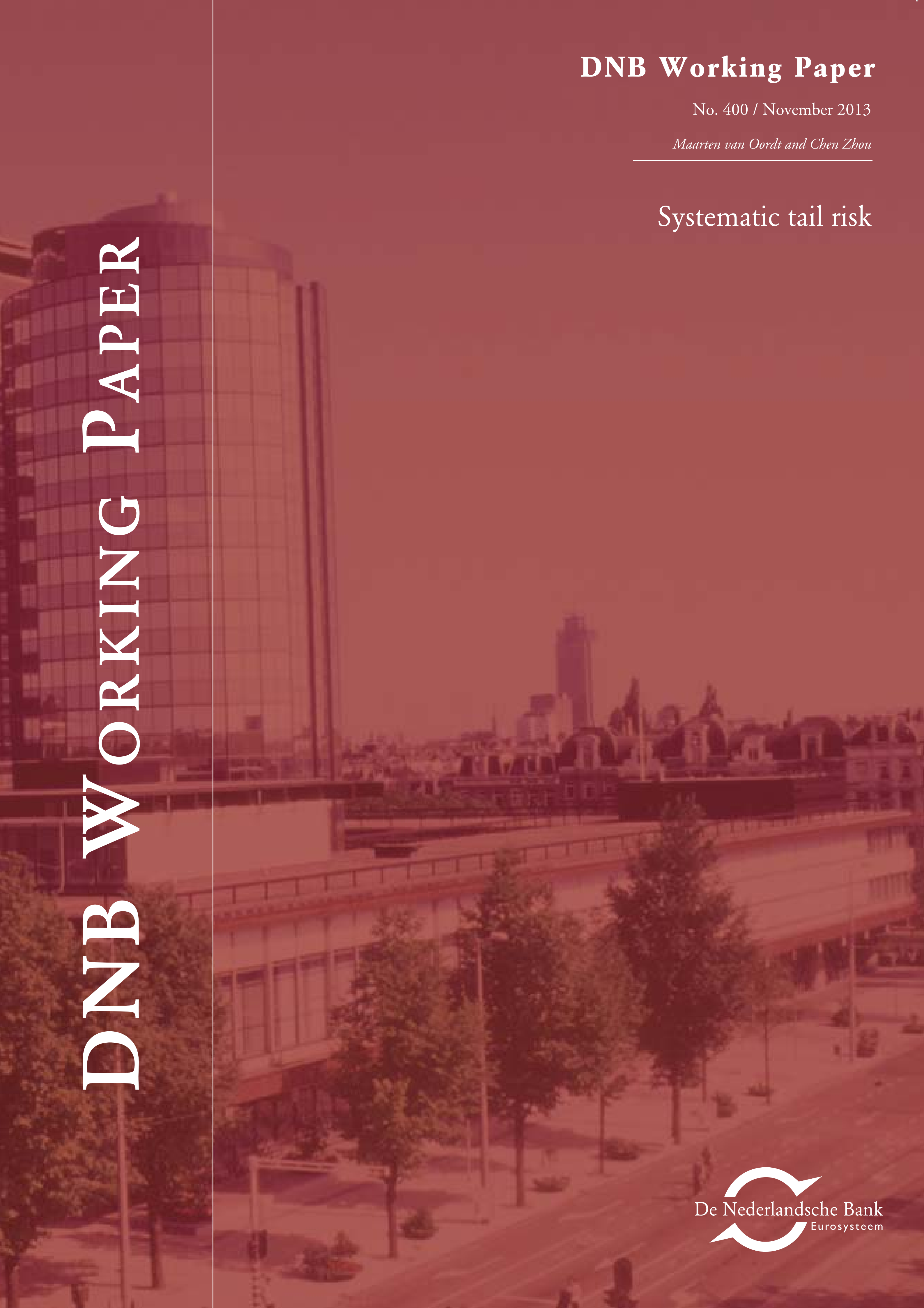

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De Nederlandsche Bank
Eurosysteem

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* Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

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De Nederlandsche Bank NV
P.O. Box 98
1000 AB AMSTERDAM
The Netherlands

Systematic tail risk*

Maarten R.C. van Oordt†‡
De Nederlandsche Bank

Chen Zhou§
De Nederlandsche Bank and Erasmus University Rotterdam

Abstract We test for the presence of a systematic tail risk premium in the cross-section of expected returns by applying a measure on the sensitivity of assets to extreme market downturns, the tail beta. Empirically, historical tail betas help to predict the future performance of stocks under extreme market downturns. During a market crash, stocks with historically high tail betas suffer losses that are approximately 2 to 3 times larger than their low tail beta counterparts. However, we find no evidence of a premium associated with tail betas. The theoretically additive and empirically persistent tail betas can help to assess portfolio tail risks.

Keywords: Tail beta, systematic risk, asset pricing, Extreme Value Theory, risk management.

JEL Classification Numbers: G11, G12

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†Tel.: +31 20 524 2916; Fax.: +31 20 524 2506; Postal address: Economics and Research Division, De Nederlandsche Bank, 1000AB Amsterdam, The Netherlands. Views expressed are those of the authors and do not necessarily reflect official positions of De Nederlandsche Bank.

‡Email: M.R.C.van.Oordt@DNB.nl

§Email: C.Zhou@DNB.nl, zhou@ese.eur.nl

1 Introduction

Risk managers are concerned with the performance of portfolios in distress events, the so-called tail events in the market. In this paper, we investigate the sensitivity of assets to market risk under extremely adverse market conditions, i.e., their loading on systematic tail risk. We estimate an additive measure of sensitivity to systematic tail risk, the 'tail beta'. We examine whether the estimated loadings on systematic tail risk help to explain the cross-section of expected returns, and discuss their potential application in risk management.

Systematic tail risk may play an important role in asset pricing. Arzac and Bawa (1977) derive an asset pricing theory under the safety-first principle of Telser (1955). They consider investors who maximise their expected return under a Value-at-Risk constraint. In their framework, the cross-section of expected returns is explained by a 'beta' that is different from the regular market beta in the Capital Asset Pricing Model (CAPM). If investors are constrained by a Value-at-Risk with a sufficiently small probability, then the aforementioned tail beta equals the beta of Arzac and Bawa (1977), assuming a linear model under extremely adverse market conditions.

Our empirical results provide evidence that historical tail betas help to assess which stocks will take relatively large hits during future market crashes. Starting with checking the persistence, we find that the persistency of the classification of firms based on either tail betas or regular market betas is comparable, even though tail betas are estimated from a few tail observations only. Based on the tail beta classification, we find that stocks with historically high tail betas suffer losses during market crashes that are on average approximately 2 to 3 times larger than their low tail beta counterparts.

Further, we test whether the estimated tail betas help to explain the cross-section of expected returns. That is, we test whether stocks with high tail betas are compensated by a risk premium. Surprisingly, from the asset pricing tests we do not observe a premium for stocks with high tail betas. We stress that this finding is not a consequence of losses suffered during the recent financial crisis. The risk premium remains absent if this episode is excluded from our sample. Hence, based on stock market data, the role of systematic tail risk in explaining the cross-section of expected returns seems to be limited.

These results are not explained by many other factors documented in the asset pricing literature and are robust against methodological alterations. The results are established within all size cohorts (Fama and French (2008)) and in the context of both equal- and value-weighted portfolios. They are robust when controlling for downside beta, coskewness, cokurtosis, idiosyncratic risk (Ang et al. (2006b) and volume (Gervais et al. (2001)). The results are not explained by return characteristics related to past performance, such as short-term reversal (Jegadeesh (1990)), momentum (Carhart (1997)) and long-term reversal (De Bondt and Thaler (1985)).

We focus on tail betas, because regular market betas do not necessarily provide an accurate description of the loading on systematic risk under all market conditions. It is a well-known stylised fact that equity returns show higher correlations during periods of high stock market volatility; see e.g. King and Wadhwani (1990), Longin and Solnik (1995), Karolyi and Stulz (1996) and Ramchand and Susmel (1998). In addition, correlations increase, especially during periods of severe market downturns as reported by Longin and Solnik (2001), Ang and Bekaert (2002), Ang and Chen (2002) and Patton (2004). This changing correlation structure may signal that the comovement of assets with the market depends on market conditions.

Several studies address the changing comovement in the context of a non-linear relation between asset returns and market risk. In line with the theoretical work of Rubinstein (1973) and Kraus and Litzenberger (1976), one strand of literature estimates the relation between asset returns and market risk with higher order approximations. Among others, Harvey and Siddique (2000) and Dittmar (2002) find that *coskewness* and *cokurtosis* play a role in asset pricing, respectively. However, room for extensions with additional higher moments may be limited, because the heavy tails observed in stock returns provide evidence that further higher moments may not exist; see Mandelbrot (1963) and Jansen and De Vries (1991).

An alternative strand of literature focuses on the comovement of asset returns with the market under specific market conditions. In line with the theory in Bawa and Lindenberg

(1977), several studies investigate the downside beta, which is defined as the market beta conditional on below average or below zero market returns; see e.g. Price et al. (1982), Harlow and Rao (1989) and Ang et al. (2006a). Our tail beta fits within this latter strand of literature, because it focuses on comovement with the market return under specific market conditions. However, in contrast to the downside beta, the tail beta measures comovement with the market if market downturns are extreme.

Our study is related to the empirical asset pricing literature on tail risks. A few studies focus on the role of tail risk in the cross-section of expected return, irrespective of its relation with market risk; see e.g. Bali et al. (2009), Huang et al. (2012) and Cholette and Lu (2011). Alternatively, Kelly (2011) constructs an index on the level of tail risks in the market and obtains 'tail risk betas' for individual assets by regressing asset returns on innovations in this index. These betas can be considered as a tail equivalent of the volatility betas of Ang et al. (2006b), which measure the comovement of stock returns with innovations in market volatility. In contrast to the 'tail risk beta' of Kelly (2011), our 'tail beta' can be considered as a tail equivalent of the market beta.

The expression 'tail beta' appears in the literature with other meanings. For example, De Jonghe (2010) estimates 'tail betas' by applying a tail dependence measure from Poon et al. (2004) on stock returns. Spitzer (2006) and Ruenzi and Weigert (2012) examine its asset pricing power. This tail dependence measure is defined as the probability of an extreme downward movement of the asset, conditional on the occurrence of a market crash. Hence, instead of measuring the magnitude of the comovement, this measure has the attributes of a conditional probability. Further, Bali et al. (2011) estimate 'hybrid tail betas'. Their aim is to capture the covariance of the asset and the market, given an adverse return on the *asset*.

Compared to these measures, the tail beta we estimate shares two appealing features with the regular market beta. First, its interpretation as a measure of comovement with the market is in absolute terms. That is, on a day that the market suffers a loss of 10%, an asset with a tail beta of 2 is expected to suffer a downward movement of 20%. Second, the tail beta we estimate is an additive measure of tail risk. In other words, the tail beta of an investment portfolio is the weighted average of the tail betas of the individual assets. Consequently, the estimated tail betas provide a clear insight into how each asset contributes to the systematic tail risk of a portfolio, which is treated in the risk management section.

2 Theory

To define the tail beta, we first introduce a linear model that decomposes asset returns under extremely adverse market conditions into a systematic and an idiosyncratic component. We denote the return on asset j and the market portfolio as R_j and R_m. The excess return on asset j and the market are given by $R_j^e = R_j - R_f$ and $R_m^e = R_m - R_f$, where R_f is the risk-free rate. The following model relates the asset excess return to large losses on the market portfolio

$$R_j^e = \beta_j^T R_m^e + \varepsilon_j, \quad \text{for} \quad R_m^e < -VaR_m(\bar{p}), \tag{2.1}$$

where ε_j denotes the idiosyncratic risk that is uncorrelated with R_m^e under the condition $R_m^e < -VaR_m(\bar{p})$ and $E\varepsilon_j = 0$. The $VaR_m(\bar{p})$ denotes the Value-at-Risk (VaR) of the excess market return with some low probability $\bar{p}$ such that $\Pr(R_m^e \leq -VaR_m(\bar{p})) = \bar{p}$; in other words, it is the loss on the market that is exceeded with probability $\bar{p}$. The parameter β_j^T measures the sensitivity to systematic tail risk and will be defined as the 'tail beta'.

The linear tail model in (2.1) specifies the comovement between the asset and the market excess return only under extremely adverse market conditions. Nevertheless, safety-first investors do not need any further assumptions to value each asset according to the asset pricing theory developed by Arzac and Bawa (1977). In particular, given the linear tail model in (2.1), we show that the beta that determines expected returns in the asset pricing theory of AB1977 is identical to the tail beta.

The asset pricing theory of AB1977 builds on the assumption that investors maximise the expected return while limiting the probability of suffering a particularly large loss below a predetermined admissible level p.[1] In other words, investors maximise the expected return under a VaR constraint. Under this objective function, AB1977 prove in a distribution-free setup that the equilibrium price for any asset j is given by

$$E(R_j^e) = \beta_j^{AB} E(R_m^e), \tag{2.2}$$

where the parameter β_j^{AB} can be derived as (see Appendix A)

$$\beta_j^{AB} = \frac{E(R_j^e | R_m^e = -VaR_m(p))}{-VaR_m(p)}. \tag{2.3}$$

The parameter β_j^{AB} in equation (2.3) is determined by the asset's contribution to the VaR of the market portfolio with a probability level equal to the admissible probability p.

Given the linear tail model in (2.1), suppose that the investors care about sufficiently large losses, such that the admissible probability p is smaller than the $\bar{p}$ in the linear tail model. We can then express the β_j^{AB} in (2.3) as

$$\begin{aligned}
\beta_j^{AB} &= \frac{E(\beta_j^T R_m^e + \varepsilon_j | R_m^e = -VaR_m(p))}{-VaR_m(p)} \\
&= \beta_j^T + \frac{E(\varepsilon_j)}{-VaR_m(p)} \\
&= \beta_j^T .
\end{aligned}$$

Hence, we obtain that the tail beta, β_j^T, is identical to the beta in the AB1977 asset pricing theory, β_j^{AB}. Consequently, given the linear tail model in (2.1), the expected returns of assets under the safety-first framework depend on their tail betas.

[1]The initial safety-first principle introduced by Roy (1952) assumes that agents minimise the probability of suffering a large loss. AB1977 adapt the formulation by Telser (1955), which assumes that agents do not want the probability of suffering a particularly large loss to exceed a pre-specified level.

3 Methodology

Our objective is to test whether assets with relatively high tail betas earn an additional systematic tail risk premium. For that purpose, we collected daily and monthly data on NYSE, AMEX and NASDAQ stocks of non-financials between July 1963 and December 2010 from the Center for Research in Security Prices. In addition, we collected the risk-free rates and the excess returns on the market portfolio from the data library section on Kenneth French's website.

We estimate firm specific tail betas at the start of each month between July 1968 and December 2010. A potential difficulty with the estimation of tail betas is the low number of observations that correspond to extremely adverse market conditions. Researchers often estimate market betas based on the past 60 monthly returns. Such a low number of observations is insufficient for our purpose of estimating tail betas. We therefore use daily returns from the past 60 months in our estimates, which corresponds to approximately 1,250 days.

We estimate tail betas using the estimation methodology based on Extreme Value Theory (EVT) developed by Van Oordt and Zhou (2011). The basic assumption of this approach is that the market and asset returns are heavy-tailed with the following expansion on the tail of their distribution functions:

$$\Pr(R_m^e < -u) \sim A_m u^{-\alpha_m} \text{ and } \Pr(R_j^e < -u) \sim A_j u^{-\alpha_j}, \text{ as } u \to \infty. \quad (3.1)$$

The parameters α_m and α_j are called the *tail indices*, and the parameters A_m and A_j are the *scales*. The idiosyncratic risk, ε_j, and the market risk, R_m^e, are assumed to be independent. The linear tail model in (2.1) induces a dependence structure between extremely adverse market returns and the asset returns. The tail beta is estimated by exploiting the tail dependence structure and using the observations in the 'tail region' only. With the number of observed returns denoted by n, only the k lowest returns are used in the estimation.[2] The estimator of the tail beta is given as

$$\hat{\beta}_j^T := \widehat{\tau_j(k/n)}^{1/\hat{\alpha}_m} \frac{\widehat{VaR_j(k/n)}}{\widehat{VaR_m(k/n)}}, \tag{3.2}$$

with four components obtained as follows. First, the tail index α_m can be estimated from the k highest market losses with the so-called *Hill estimator* proposed by Hill (1975).[3] Consider the losses $X_t^{(m)} = -R_{m,t}^e$, for $t = 1, \cdots, n$. By ranking them as $X_{n,1}^{(m)} \leq X_{n,2}^{(m)} \leq \cdots \leq X_{n,n}^{(m)}$, the Hill estimator is calculated as

$$\frac{1}{\hat{\alpha}_m} = \frac{1}{k} \sum_{i=1}^{k} \log X_{n,n-i+1}^{(m)} - \log X_{n,n-k}^{(m)}. \tag{3.3}$$

Second, the $\tau_j(k/n)$ parameter can be estimated non-parametrically by a counting measure

$$\widehat{\tau_j(k/n)} := \frac{1}{k} \sum_{t=1}^{n} \mathbf{1}_{\left\{X_t^{(j)} > X_{n,n-k}^{(j)} \text{ and } X_t^{(m)} > X_{n,n-k}^{(m)}\right\}}, \tag{3.4}$$

where $X_{n,n-k}^{(j)}$ is the $(k+1)-$th highest loss on the asset, and where $X_t^{(j)} := -R_{j,t}^e$, $t = 1, \cdots, n$. This parameter characterises the tail dependence between the market and the asset; see e.g. De Haan and Ferreira (2006). Finally, the VaRs of the market and asset return at probability level k/n are estimated by their $(k+1)-$th highest losses.[4]

At the end of every estimation window we rank the firms based on their tail betas and construct five portfolios, each of which contains the same number of stocks. To

[2]Theoretically, the EVT approach defines $k := k(n)$ as an intermediate sequence such that $k := k(n) \to \infty$ and $k/n \to 0$ as $n \to +\infty$. In practice, these conditions on k are not relevant for a finite sample size n. For low values of k, the estimate exhibits a large variance, while for high values of k, it bears a potential bias, because observations from relatively regular market conditions are included in the estimation. Practically, we choose $k = 50$ days in each estimation window of 60 months, which corresponds to a k/n-ratio of roughly 4%. The results are robust if tail betas are estimated with $k = 30$.

[3]The EVT approach needs the weak condition that $\alpha_j > \frac{1}{2}\alpha_m$. This condition requires a lower bound on the tail index of asset excess returns. Empirical research usually finds that α_m is around 4; see e.g. Jansen and De Vries (1991), Loretan and Phillips (1994) and Poon et al. (2004). In line with these results, we observe $\hat{\alpha}_m = 3.5$ as an average estimate for the market. Given these findings, the condition is equivalent to $\alpha_j > 2$, which is satisfied if the excess returns of individual assets have finite variance.

[4]An alternative approach to estimate tail betas involves performing a regression on the observations corresponding to the k largest market losses. For example, Post and Versijp (2007) provide estimates of tail beta from regressions conditional on market returns below -10%. Our results are robust for using the conditional regression approach. However, the estimates from this approach yield a less persistent ranking of firms over time and result in a smaller return difference between high and low tail beta stocks in case of extreme market downturns.

maximise the potential variation after controlling for regular market risk, we also sort stocks based on their tail beta spreads, i.e., the spread between tail betas and regular market betas.[5] In the portfolio formation procedure we exclude firms that do not qualify according to the following two conditions. First, stocks should not report zero returns on more than 60% of the trading days in the estimation window. We use this criterion to avoid our results being distorted by daily returns of thinly traded stocks. Second, the stock must be trading at a price above 5 USD on the last day of the estimation period. We use this criterion to exclude penny stocks that potentially represent firms in severe financial distress.[6] In summary, portfolio formation occurs at the start of each month using estimates based on returns from the past 60 months. The holding period is the first month after the estimation window.

After constructing the portfolios, we calculate monthly portfolio returns. The excess return is calculated by averaging the excess returns on individual stocks in each portfolio using both equal (EW) and value weights (VW). Further, using several benchmark models, we calculate risk-adjusted returns for individual stocks as follows:

$$R^*_{j,t} = R_{j,t} - R_{f,t} - \sum_{k=1}^{m} \hat{\beta}_{j,k} F_{k,t}, \tag{3.5}$$

where $R_{j,t}$ is the monthly return on stock j at time t, $R_{f,t}$ is the risk-free rate, and $F_{k,t}$ denotes the of k-th of m risk factors in the benchmark model. We estimate the factor loadings, $\hat{\beta}_{j,k}$ for individual stocks using regressions on monthly returns in the 60-month estimation window preceding t. The risk-adjusted returns of the tail beta spread portfolios are calculated by averaging the risk-adjusted returns of the individual stocks in each portfolio. Based on the constructed portfolio returns, we then construct the zero-investment portfolio, which is obtained by taking a long position of 1 USD in the portfolio with the 20% highest tail beta spreads, while taking a short position of 1 USD in the portfolio with the 20% lowest spreads.

[5] In this respect we follow Ang et al. (2006a), who sort based on the spread between downside beta and market beta.

[6] Empirically, historical tail betas do not provide much information about the performance of penny stocks in future extreme market downturns. Including penny stocks in the analysis does not qualitatively affect our conclusions.

4 Results

4.1 Descriptive statistics

Table 1 reports the average of some descriptive statistics across the stocks in each of the sorted portfolios. When sorting according to the estimated tail betas in panel (a), we observe that stocks with high tail betas also tend to have high market betas on average. There is a clear trend in the average market betas across the tail beta portfolios. Trends with similar signs can be observed for the downside beta, the standard deviation, the skewness and the excess kurtosis of the returns. These trends indicate that stocks with high tail betas also tend to have higher values for other potential risk measures. Interestingly, the tail dependence measure, τ, which is an ingredient of the tail beta calculation, remains on average at a relatively constant level across the different portfolios. Apparently, there is not a very strong relation between the firms that comove more with the market in extremely adverse market conditions, and those firms that have a high level of tail dependence. Finally, high tail beta firms are relatively small in terms of market capitalization, but tend to have higher trading volumes.

Most trends fade or reverse if we sort on the spread between the tail beta and the market beta in panel (b). Apparently, many of the aforementioned trends are rather driven by high regular market betas than by high tail betas. After sorting on the spread we observe that both the market beta and the downside beta increase as the spread decreases. Further, a clear trend appears for coskewness. However, no strong trend can be observed for the other return characteristics. The trend in trading volume is also reversed: high spread firms are less frequently traded. The relation between the tail beta and the market beta stress the importance of sorting on the spread between tail beta and market beta. Moreover, the relation between the tail beta and other risk measures, especially downside beta and coskewness, stresses the importance of performing robustness checks in our asset pricing tests.

Table 1: Descriptive statistics

Panel (a): sorting on β^T	High β^T	4	3	2	Low β^T
Return characteristics:					
$\bar{\beta}$	1.70	1.32	1.10	0.91	0.62
$\overline{\beta^T - \beta}$	0.91	0.62	0.48	0.37	0.27
$\bar{\beta}^{DS}$	1.72	1.33	1.11	0.92	0.65
Standard deviation	18.30	13.70	11.12	9.33	7.40
Idiosyncratic volatility	15.20	11.40	9.25	7.77	6.34
$\bar{\tau}$	0.22	0.21	0.21	0.20	0.15
Skewness	0.81	0.52	0.38	0.31	0.35
Coskewness	-0.04	-0.04	-0.04	-0.04	-0.03
Excess kurtosis	2.55	1.80	1.52	1.38	1.65
Cokurtosis	0.03	0.06	0.09	0.11	0.10
Firm characteristics:					
Market capitalization (bln USD)	0.74	1.19	2.15	3.02	2.66
Volume (mln shares)	10.21	8.05	7.97	7.74	5.25

Panel (b): sorting on $\beta^T - \beta$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$
Return characteristics:					
$\bar{\beta}$	0.91	1.01	1.05	1.16	1.54
$\overline{\beta^T - \beta}$	1.32	0.72	0.47	0.25	-0.11
$\bar{\beta}^{DS}$	1.08	1.05	1.05	1.13	1.42
Standard deviation	14.66	11.58	10.54	10.49	12.59
Idiosyncratic volatility	13.03	9.89	8.76	8.48	9.81
$\bar{\tau}$	0.19	0.21	0.21	0.20	0.18
Skewness	0.71	0.44	0.37	0.36	0.51
Coskewness	-0.10	-0.06	-0.03	-0.02	0.02
Kurtosis	2.22	1.51	1.46	1.53	2.18
Cokurtosis	0.11	0.05	0.05	0.07	0.10
Firm characteristics:					
Market capitalization (bln USD)	0.79	2.03	2.60	2.52	1.81
Volume (mln shares)	4.69	7.30	8.24	8.68	10.32

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas of NYSE, AMEX and NASDAQ stocks by applying the EVT approach in (3.2) on daily returns from the past 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We exclude stocks with more than 60% zero daily returns in the 60 months prior to t, and stocks with a price below 5 USD at the end of month $t-1$. Stocks are sorted into five quintiles: panel (a) considers quintiles sorted on tail betas, panel (b) considers quintiles sorted on the spread between the tail beta and the market beta. The reported numbers are averages for the stocks in each sort. We first average across firms in each month t, and then average over the 510 months in the sample. The market beta, downside beta, standard deviation, idiosyncratic volatility, skewness, coskewness, excess kurtosis and cokurtosis are calculated from the 60 monthly returns prior to t. The market beta is reported as $\bar{\beta}$. The spread between tail beta and market beta is reported as $\overline{\beta^T - \beta}$. Downside beta, $\bar{\beta}^{DS}$, is estimated by a regression conditional on below average market returns. Idiosyncratic volatility is calculated as the standard deviation of the residuals obtained from regressing individual stock returns on the FF3 factors. The tail dependence measure, τ, is based on daily returns from the 60 months prior to t and calculated following the estimator in equation (3.4), with $k = 50$. Coskewness and Cokurtosis are calculated following (5.1) and (5.2). Market capitalization and trading volume are provided at the end of month $t-1$.

4.2 Persistence

First, we verify whether the estimates of tail beta obtained from historical data are persistent over time. In the absence of such persistence, estimating tail betas based on historical data would merely have a descriptive function, and would provide no insight in future comovements during adverse market conditions. To investigate this issue we provide transition matrices in Table 2. The transition matrices in Table 2, panel (a) are based on firm tail betas and respectively their 12 month lagged value. One concern is that the potential persistence observed in transition matrices based on 12 month lagged values is spurious because those transition matrices are based on tail beta estimates from two overlapping data samples. To address this issue, we also construct transition matrices based on 60 months lagged data samples in Table 2, panel (b). Tail betas are estimated following both the EVT approach and the conditional regression approach. The table also provides a similar matrix for market betas estimated from a regression with the CAPM as benchmark model. Higher numbers along and around the diagonal point into the direction of a more persistent sorting.

In Table 2 we observe two patterns from the transition matrices based on 12 month lagged values. First, the numbers along the diagonal of the transition matrices are higher if the matrix is constructed from tail betas estimated with the EVT approach. This suggests that the EVT approach provides a more persistent classification of firms based on the sensitivity to systematic tail risk than the conditional regression approach. The higher persistence is a potential consequence of the lower variance associated with the EVT approach. Second, the numbers along and around the diagonal of the transition matrices based on tail beta are in general above those in the transition matrix constructed from market betas. This pattern suggest that if one is inclined to believe that historical market betas contain information about future comovement with the market, then there seems to be no reason to worry about the information contained by historical tail betas on future comovement with the market under extremely adverse market conditions. Not surprisingly, the overall level of the numbers on the diagonal is lower in the transition matrices based on 60 month lagged values. However, the observed patterns remain: tail

Table 2: Transition matrices

Panel (a): 12 months			t + 12				
EVT approach			High β^T	4	3	2	Low β^T
		High β^T	80	18	2	0	0
		4	15	62	21	2	0
	t	3	2	16	59	21	2
		2	0	2	17	64	16
		Low β^T	0	1	2	14	83
Conditional approach			High β^T	4	3	2	Low β^T
		High β^T	74	18	5	2	1
		4	18	52	21	7	2
	t	3	5	20	48	22	6
		2	2	7	21	50	20
		Low β^T	1	2	6	20	71
Market beta			High β	4	3	2	Low β
		High β	76	20	3	1	0
		4	17	54	23	5	1
	t	3	3	21	51	22	3
		2	1	4	21	56	18
		Low β	0	1	3	17	79

Panel (b): 60 months			t + 60				
EVT approach			High β^T	4	3	2	Low β^T
		High β^T	49	29	14	6	2
		4	21	30	27	17	6
	t	3	9	18	30	30	13
		2	4	11	23	35	26
		Low β^T	2	4	10	22	62
Conditional approach			High β^T	4	3	2	Low β^T
		High β^T	34	24	18	14	10
		4	21	23	21	19	15
	t	3	15	20	22	23	20
		2	12	18	22	24	25
		Low β^T	8	14	19	25	33
Market beta			High β	4	3	2	Low β
		High β	37	25	17	11	6
		4	20	26	25	19	10
	t	3	12	22	26	25	15
		2	8	15	24	28	25
		Low β	4	8	14	26	52

Note: The table provides transition matrices based on 12 month (panel a) and 60 month (panel b) lagged values. At the start of each month t between July 1968 and Dec 2009, we estimate tail betas of NYSE, AMEX and NASDAQ stocks by applying the EVT approach in (3.2) on past daily returns from the 60 months prior to t. Stocks are sorted into five quintiles according to the tail beta estimates. We exclude stocks with more than 60% zero daily returns in the 60 months prior to t, and stocks with a price below 5 USD at the end of month $t-1$. We also determine the allocation of each firm in the tail beta quintiles at the start of month $t+12$. For each tail beta quintile at time t we calculate the percentage of surviving firms allocated in each tail beta quintile at the start of month $t+12$. The results in the transition matrices are averages over time.
We repeat the procedure for tail betas obtained from the conditional regression approach by performing a regression on daily returns conditional on the 50 worst market returns from the 60 months prior to t. We also repeat the procedure for market betas obtained from a regression on monthly returns from the 60 months prior to t with the CAPM as benchmark model. The lower panel provides the same matrices after sorting at the start of month t (between July 1968 and Dec 2005) and at the start of month $t+60$. Higher numbers along and around the diagonal of the transition matrices point into the direction of a more persistent sorting.

betas from the EVT approach provide a more persistent sorting of firms; and the sorting of firms based on tail beta is not less persistent than the sorting of firms on market beta.

4.3 Asset pricing tests

Table 3 presents the baseline asset pricing result on the sorted portfolios. The unadjusted average excess return on the zero-investment portfolio is slightly below, but not significantly different from zero. For returns adjusted with the CAPM and the Fama and French (1993) three factor model (FF3), the return on the zero-investment portfolio is slightly above, but still not significantly different from zero. This result holds for both equal and value weighted portfolios (the t-statistics are between 0.6 and 1.0). Also if we repeat our procedure within presorted size cohorts and calculate the FF3-adjusted returns, the absence of any significant risk premium remains prevalent. To summarize, we cannot reject the null hypothesis that having an exposure on systematic tail risk did not receive an additional (ex post) risk premium in the market.

The results so far are rather pessimistic about the role of systematic tail risk in explaining the cross-section of expected returns. One potential reason for not finding any significant results might be the failure to reliably estimate a forward looking measure of the sensitivity to systematic tail risk. The definition of tail beta only implies that high tail beta stocks suffer from large in-sample losses under extremely adverse market conditions. Nevertheless, the results in the transition matrices seem to suggest that the sorting based on estimates of tail beta is quite persistent over time. This persistency hints, but does not guarantee that historical tail betas capture future sensitivity towards systematic tail risk. To test this explicitly, we take the (risk-adjusted) portfolio returns from the last exercise, and focus on the months with an excess market return, $R^e_{m,t}$, lower than -5%. We consider these 54 months as representing 'extremely adverse market conditions'.

Table 4 reports the results for the subsample of months with extremely adverse market conditions. The first line in Table 4 reports the average historical returns on portfolios sorted on tail beta alone. Across the portfolios, we observe a strong downward sloping trend in the average losses if one moves from the portfolios with the highest tail betas

Table 3: General asset pricing results

	Sort:		High β^T	4	3	2	Low β^T	(5) - (1)
$\bar{R}^e_p$	EW		0.42	0.72	0.75	0.70	0.66	-0.24
			(1.1)	(2.3)	(2.8)	(3.0)	(3.6)	(-0.8)
	VW		0.37	0.53	0.45	0.40	0.50	-0.13
			(0.9)	(1.6)	(1.8)	(1.9)	(3.1)	(-0.4)
	Sort:		High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
$\bar{R}^e_p$	EW		0.56	0.69	0.67	0.69	0.65	-0.09
			(1.8)	(2.6)	(2.7)	(2.7)	(2.2)	(-0.6)
	VW		0.44	0.53	0.48	0.45	0.44	-0.01
			(1.5)	(2.3)	(2.3)	(2.1)	(1.7)	(-0.0)
$\bar{R}^*_{CAPM,p}$	EW		0.24	0.31	0.27	0.24	0.04	0.20
			(1.4)	(2.8)	(3.0)	(2.3)	(0.2)	(0.9)
	VW		0.11	0.19	0.13	0.02	-0.12	0.22
			(0.7)	(2.3)	(2.5)	(0.3)	(-1.0)	(1.0)
$\bar{R}^*_{FF3,p}$	EW		0.03	0.10	0.05	0.03	-0.09	0.11
			(0.3)	(2.3)	(1.2)	(0.6)	(-0.8)	(0.6)
	VW		0.11	0.17	0.08	0.02	-0.03	0.14
			(0.9)	(2.4)	(1.4)	(0.4)	(-0.3)	(0.7)
$\bar{R}^*_{FF3,p}$	EW	Small	-0.15	0.14	0.18	0.06	0.05	-0.19
		2	-0.03	0.03	0.11	0.06	-0.08	0.05
		3	0.09	0.03	0.05	0.05	-0.10	0.19
		4	0.06	0.13	0.07	0.00	-0.10	0.16
		Large	0.14	0.04	0.01	-0.09	-0.08	0.22
		Avg	0.02	0.07	0.08	0.02	-0.06	0.08
t-stat		Small	(-0.9)	(1.3)	(2.1)	(0.7)	(0.4)	(-0.8)
		2	(-0.3)	(0.4)	(1.4)	(0.7)	(-0.5)	(0.2)
		3	(0.7)	(0.5)	(0.8)	(0.6)	(-0.8)	(0.9)
		4	(0.5)	(1.7)	(1.2)	(-0.0)	(-0.8)	(0.9)
		Large	(1.5)	(0.8)	(0.1)	(-1.4)	(-0.8)	(1.3)
		Avg	(0.2)	(1.6)	(1.9)	(0.3)	(-0.6)	(0.5)
$\bar{R}^*_{FF3,p}$	VW	Small	-0.23	0.11	0.14	0.03	0.04	-0.27
		2	-0.04	-0.01	0.09	0.06	-0.08	0.03
		3	0.11	0.06	0.04	0.04	-0.11	0.22
		4	0.06	0.14	0.06	-0.02	-0.09	0.14
		Large	0.17	0.15	0.05	-0.09	-0.03	0.20
		Avg	0.01	0.09	0.08	0.00	-0.05	0.07
t-stat		Small	(-1.4)	(1.2)	(1.6)	(0.3)	(0.3)	(-1.2)
		2	(-0.4)	(-0.1)	(1.1)	(0.6)	(-0.5)	(0.1)
		3	(0.9)	(0.8)	(0.5)	(0.5)	(-0.8)	(1.0)
		4	(0.5)	(2.0)	(1.0)	(-0.2)	(-0.7)	(0.8)
		Large	(1.8)	(2.0)	(0.8)	(-1.3)	(-0.2)	(1.1)
		Avg	(0.1)	(1.9)	(1.8)	(0.1)	(-0.5)	(0.4)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (3.2) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 equal weighted (EW) and value weighted (VW) portfolios by sorting on either the tail beta (the first row) or the spread between tail beta and market beta (from the second row onwards) and construct a zero-investment portfolio. We calculate risk-adjusted returns by applying (3.5) on monthly stock returns at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t.
The first and second row report the average excess portfolio return, $\bar{R}^e_p$. The third and fourth rows, report the average CAPM- and FF3-adjusted portfolio returns, $\bar{R}^*_{CAPM,p}$ and $\bar{R}^*_{FF3,p}$. The fifth and sixth rows report the average FF3-adjusted returns after presorting the equities in five size cohorts and then sorting on the tail beta spread within each size cohort, where size is measured by market capitalization at the end of month $t-1$. Newey-West t-statistics are reported in parentheses.

Table 4: Results under extremely adverse market conditions ($R^e_{m,t} < -5\%$)

	Sort:		High β^T	4	3	2	Low β^T	(5) - (1)
$\bar{R}^e_p$	EW		-13.62	-10.20	-8.39	-7.03	-4.94	-8.68
			(-14.9)	(-12.4)	(-10.9)	(-10.2)	(-8.8)	(-11.8)
	VW		-14.22	-11.46	-9.17	-7.13	-5.12	-9.10
			(-15.0)	(-15.6)	(-15.7)	(-13.4)	(-10.4)	(-9.5)
	Sort:		High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
$\bar{R}^e_p$	EW		-9.93	-8.49	-8.08	-8.26	-9.42	-0.51
			(-11.2)	(-11.0)	(-12.1)	(-12.6)	(-13.5)	(-0.9)
	VW		-9.58	-7.59	-7.69	-8.10	-9.26	-0.33
			(-13.0)	(-12.2)	(-16.6)	(-16.8)	(-15.9)	(-0.4)
$\bar{R}^*_{CAPM,p}$	EW		-2.38	-0.07	0.76	1.64	3.70	-6.07
			(-3.9)	(-0.2)	(2.3)	(4.8)	(8.1)	(-8.7)
	VW		-3.18	-1.03	-0.36	0.67	2.63	-5.81
			(-7.3)	(-3.1)	(-1.7)	(2.7)	(6.8)	(-8.7)
$\bar{R}^*_{FF3,p}$	EW		-2.52	-0.31	0.44	1.19	3.17	-5.69
			(-7.8)	(-1.7)	(3.3)	(6.1)	(9.2)	(-9.5)
	VW		-2.51	-0.71	-0.17	0.78	2.40	-4.90
			(-6.2)	(-2.3)	(-0.8)	(3.3)	(6.4)	(-8.0)
$\bar{R}^*_{FF3,p}$	EW	Small	-3.83	-1.07	0.66	1.36	3.59	-7.43
		2	-3.15	-0.53	0.31	1.35	3.91	-7.05
		3	-2.24	-0.52	0.57	1.20	3.16	-5.40
		4	-1.54	-0.08	0.46	1.34	2.90	-4.44
		Large	-1.52	-0.18	0.56	0.84	2.34	-3.87
		Avg	-2.46	-0.48	0.51	1.22	3.18	-5.64
t-stat		Small	(-8.3)	(-3.1)	(2.7)	(5.1)	(9.1)	(-10.4)
		2	(-7.1)	(-1.9)	(1.4)	(4.6)	(8.2)	(-9.0)
		3	(-5.5)	(-2.1)	(2.4)	(4.9)	(8.2)	(-7.8)
		4	(-4.1)	(-0.3)	(2.0)	(5.2)	(7.5)	(-7.4)
		Large	(-4.5)	(-0.9)	(2.7)	(3.3)	(6.5)	(-7.6)
		Avg	(-7.6)	(-2.8)	(3.7)	(6.3)	(9.3)	(-9.6)
$\bar{R}^*_{FF3,p}$	VW	Small	-3.92	-1.16	0.58	1.35	3.54	-7.45
		2	-3.15	-0.45	0.25	1.38	3.87	-7.02
		3	-2.15	-0.44	0.64	1.20	3.14	-5.29
		4	-1.57	-0.13	0.51	1.34	2.87	-4.44
		Large	-1.59	-0.12	-0.08	0.48	2.30	-3.89
		Avg	-2.48	-0.46	0.38	1.15	3.14	-5.62
t-stat		Small	(-8.7)	(-3.4)	(2.5)	(4.9)	(9.1)	(-10.7)
		2	(-7.4)	(-1.6)	(1.1)	(4.7)	(8.2)	(-9.1)
		3	(-5.3)	(-1.8)	(2.5)	(4.7)	(8.2)	(-7.9)
		4	(-4.1)	(-0.5)	(2.2)	(5.3)	(7.6)	(-7.4)
		Large	(-4.0)	(-0.5)	(-0.3)	(1.8)	(5.7)	(-6.4)
		Avg	(-8.1)	(-2.8)	(3.0)	(6.3)	(9.5)	(-9.9)

Note: We calculate the average of the excess returns and risk-adjusted returns conditional on extremely adverse market conditions for the portfolios sorted on tail beta (the first row) or the spread between the tail beta and the market beta (from the second row onwards). From July 1968 and Dec 2010, we consider the returns from the 54 months in which the market factor lost at least 5% of its value, i.e., $R^e_{m,t} < -5\%$.

The first row and second row report the average excess portfolio return, $\bar{R}^e_p$. The third and fourth row, report the CAPM- and FF3-adjusted portfolio return, $\bar{R}^*_{CAPM,p}$ and $\bar{R}^*_{FF3,p}$. The fifth and sixth row report the average FF3-adjusted returns after presorting the equities in five size cohorts based on market capitalization at the end of month $t-1$ and then sorting on the tail beta spread within each size cohort. Standard t-statistics are reported in parentheses.

to those with low tail betas. For the high tail betas we find an average loss of 13.62%, while the average loss for the portfolio with the lowest tail beta loadings equals to 4.94%. However, this result may merely reflect the underlying difference in terms of the (regular) market betas. Consequently, we continue to the portfolios sorted on the spread between tail beta and market beta.

Considering sorts based on the spread between tail and market beta, the difference in average losses, $\bar{R}_p^e$, seems to disappear. However, from the descriptive statistics we know that portfolios with a relatively high spread tend to have a low market beta. Consequently, we consider the risk-adjusted returns using the CAPM as benchmark model, $\bar{R}^*_{CAPM,p}$. We find that portfolios with a high positive spread between tail beta and market beta strongly underperform during extremely adverse months, while portfolios with a negative spread between tail beta and market beta strongly outperform. The zero investment portfolio, with a long exposure on high spread stocks and a short exposure on low spread stocks, yields significant additional losses during extremely adverse months. The CAPM-adjusted returns during extremely adverse months add up to -6.07% and -5.81% for respectively the value and equal weighted portfolios (t-statistics around -8.7). The FF3-adjusted returns do report similar results. The trend in the losses and the significance of the results are also robust among all size cohorts. The least significant t-statistic for the zero investment portfolios is -6.4 for the value weighted portfolio based on large firms. To summarize, from the results in Table 4 we find that stocks with a high spread between tail beta and market beta strongly underperform during extremely adverse market conditions. Hence, it seems implausible that the failure to establish a risk premium on tail beta stems from its potential failure in capturing the future sensitivity to systematic tail risk.

Another potential reason for not observing the additional risk premium for systematic tail risk comes from the double-edged sword impact of high tail betas. Although investors may receive a premium during good times, large losses are suffered during extreme market downturns. These large negative returns may partly cancel out the positive risk premium. Hence, to test for the presence of a positive risk premium during the 'business as usual' months, we also exclude months that coincide with extremely adverse market conditions.

Table 5 reports the baseline results for the subsample with the remaining 457 months. Based on these results, we find that stocks with a high spread between tail and market beta, do significantly outperform low spread stocks during 'business as usual' periods. The average risk-adjusted premium with FF3 as benchmark model is 0.80% and 0.74% per month for respectively the equal and value weighted portfolios (with t-statistics of 5.6 and 4.3). Although these results are weaker for the smallest firms, the results are robust and significant among all size cohorts.

The asset pricing tests show that investors receive a significant premium for having higher loadings on systematic tail risk during normal times. However, historically the premium is barely enough to compensate for the additional losses that occur during extremely adverse market conditions. This seems to be a reason why an additional risk premium on systematic tail risk is not observed over the entire historical sample. To conclude, from our results the additional role of systematic tail risk in explaining the cross-section of expected return seems to be limited.

5 Robustness checks

In this section we test whether our results are robust, and, in particular, whether other findings on asset pricing factors in the literature can explain part of our results. In the first subsection, we extend the FF3 benchmark model with several other factors that are relevant in the empirical asset pricing literature. Because Daniel and Titman (1997) find evidence that return premia on stock characteristics are not necessarily due to loadings on pervasive risk factors, we also check whether the findings on tail beta remain robust after presorting on several stock characteristics. We report these results in the second subsection. The final subsection provides robustness checks for deviations in the methodology.[7]

[7] Throughout the robustness checks, we report the results for the value weighted portfolios. For most robustness checks the t-statistics for the equal weighted zero investment portfolios are above those for the value weighted portfolios. Consequently, the reported significance of the premium during normal times and the significance of the loss during adverse market conditions can usually be considered to be at the conservative side.

Table 5: Results under usual market conditions ($R^e_{m,t} \geq -5\%$)

	Sort:		High β^T	4	3	2	Low β^T	(5) - (1)
$\bar{R}^e_p$	EW		2.09	2.02	1.83	1.62	1.33	0.76
			(6.4)	(8.4)	(9.1)	(9.5)	(10.2)	(2.9)
	VW		2.10	1.95	1.59	1.29	1.17	0.93
			(6.3)	(7.6)	(8.0)	(7.8)	(8.8)	(3.1)
	Sort:		High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
$\bar{R}^e_p$	EW		1.80	1.78	1.71	1.74	1.84	-0.04
			(7.4)	(8.8)	(9.2)	(9.1)	(8.4)	(-0.3)
	VW		1.62	1.49	1.45	1.46	1.59	0.03
			(7.2)	(8.3)	(9.0)	(8.4)	(7.8)	(0.2)
$\bar{R}^*_{CAPM,p}$	EW		0.55	0.35	0.22	0.08	-0.40	0.94
			(3.6)	(3.5)	(2.6)	(0.9)	(-3.2)	(6.1)
	VW		0.50	0.33	0.19	-0.06	-0.44	0.94
			(3.8)	(4.3)	(3.6)	(-1.0)	(-4.6)	(5.0)
$\bar{R}^*_{FF3,p}$	EW		0.33	0.15	0.01	-0.10	-0.47	0.80
			(3.4)	(3.1)	(0.2)	(-2.0)	(-5.3)	(5.6)
	VW		0.42	0.28	0.11	-0.07	-0.32	0.74
			(3.4)	(3.8)	(2.1)	(-1.3)	(-3.6)	(4.3)
$\bar{R}^*_{FF3,p}$	EW	Small	0.29	0.28	0.12	-0.09	-0.37	0.66
		2	0.34	0.10	0.09	-0.09	-0.55	0.89
		3	0.36	0.10	-0.01	-0.09	-0.49	0.85
		4	0.24	0.15	0.03	-0.16	-0.46	0.70
		Large	0.34	0.07	-0.06	-0.19	-0.37	0.70
		Avg	0.31	0.14	0.03	-0.13	-0.45	0.76
t-stat		Small	(2.0)	(2.7)	(1.2)	(-0.9)	(-3.0)	(3.8)
		2	(2.5)	(1.1)	(1.0)	(-1.1)	(-4.7)	(4.9)
		3	(3.0)	(1.3)	(-0.1)	(-1.2)	(-4.5)	(4.9)
		4	(2.3)	(2.0)	(0.4)	(-2.1)	(-4.2)	(4.5)
		Large	(3.8)	(1.2)	(-1.1)	(-3.1)	(-4.1)	(5.0)
		Avg	(3.4)	(2.8)	(0.7)	(-2.4)	(-5.1)	(5.4)
$\bar{R}^*_{FF3,p}$	VW	Small	0.21	0.26	0.09	-0.13	-0.37	0.59
		2	0.32	0.04	0.07	-0.10	-0.54	0.87
		3	0.37	0.12	-0.03	-0.10	-0.50	0.87
		4	0.25	0.17	0.01	-0.18	-0.44	0.68
		Large	0.38	0.18	0.06	-0.15	-0.30	0.68
		Avg	0.31	0.15	0.04	-0.13	-0.43	0.74
t-stat		Small	(1.4)	(2.5)	(0.9)	(-1.3)	(-3.0)	(3.1)
		2	(2.4)	(0.5)	(0.8)	(-1.1)	(-4.7)	(4.7)
		3	(3.0)	(1.4)	(-0.4)	(-1.4)	(-4.6)	(5.0)
		4	(2.3)	(2.2)	(0.2)	(-2.2)	(-4.0)	(4.3)
		Large	(4.2)	(2.8)	(1.1)	(-2.4)	(-3.2)	(4.4)
		Avg	(3.4)	(3.2)	(0.8)	(-2.5)	(-4.9)	(5.2)

Note: For the portfolios sorted on tail beta (the first row) or the spread between the tail beta and the market beta (from the second row onwards), we calculate average excess returns and average risk-adjusted returns conditional on usual market conditions. From July 1968 and Dec 2010, we consider the returns from the 456 months in which the market factor lost at most 5% of its value, i.e., $R^e_{m,t} \geq -5\%$. The first row and second row report the average excess portfolio return, $\bar{R}^e_p$. The third and fourth row, report the CAPM- and FF3-adjusted portfolio return, $\bar{R}^*_{CAPM,p}$ and $\bar{R}^*_{FF3,p}$. The fifth and sixth row report the average FF3-adjusted returns after presorting the equities in five size cohorts based on market capitalization at the end of month $t-1$ and then sorting on the tail beta spread within each size cohort. Standard t-statistics are reported in parentheses.

Table 6: Summary of factors in the robustness checks

Factors	Start	End	Obs	Average return	St.dev.	Source
Market	196307	201012	570	0.45	4.53	K. French
SMB	196307	201012	570	0.27	3.17	K. French
HML	196307	201012	570	0.40	2.94	K. French
Momentum	196307	201012	570	0.72	4.35	K. French
Short-term Reversal	196307	201012	570	0.54	3.16	K. French
Long-term Reversal	196307	201012	570	0.33	2.54	K. French
Liquidity	196801	201012	516	0.49	3.55	R. Stambaugh
Downside beta	196807	201012	510	0.03	3.48	Calculations
Spread ($\beta^{DS} - \beta$)	196807	201012	510	0.10	1.60	Calculations
Coskewness	196807	201012	510	-0.21	1.69	Calculations
Cokurtosis	196807	201012	510	0.02	1.79	Calculations

Note: For each risk factor in the robustness checks we provide some summary statistics. First we report the start and the end date of its availability, and the number of observations. Then we report the average excess return and its standard deviation. The last column reports the source of the risk factor. 'K. French' and 'R. Stambaugh' refer to the personal homepages of Kenneth French and Robert Stambaugh. 'Calculations' refer to the following procedure.
At the start of each month t between July 1968 and Dec 2010, we estimate the relevant risk measure for all NYSE, AMEX and NASDAQ stocks based on monthly returns from the past 60 months prior to t. Downside beta is estimated by a regression conditional on below average market returns. Coskewness and Cokurtosis are calculated following (5.1) and (5.2). We exclude stocks that have more than 60% zero daily returns in the 60 months prior to t and stocks with a price below 5 USD at the end of month $t-1$. Stocks are sorted based on the relevant risk measure. For downside beta (spread) and cokurtosis, the return on the risk factor in month t is given by calculating the value weighted return of stocks with estimates above the 70th percentile and subtract the value weighted return of stocks below the 30th percentile in month t. For coskewness, we apply the same percentile rule, but subtract the return on the portfolio with high coskewness from the return on the portfolio with low coskewness.

5.1 Extending the benchmark model

We extend the FF3 benchmark model with several other asset pricing factors documented in the literature. Table 6 reports some summary statistics and the sources of these factors and the period over which each factor is available. The results of the robustness checks are reported in Table 7 and Table 8. For each extended benchmark model we report three lines of results: we report the average risk-adjusted return of the portfolios, the average risk-adjusted return during adverse market conditions and the average risk-adjusted return during usual periods.

In the asset pricing literature several alternative characteristics are used to document potential nonlinearities in the relation between the asset and market return, such as downside beta, coskewness and cokurtosis. First, we extend the FF3 benchmark model with factors based on those nonlinearities. We construct corresponding risk factors using the following methodology. In correspondence with our estimation window of market beta and tail beta, all return characteristics necessary to construct the risk factors are calculated from returns during the past 60 months. Downside beta, β_j^{DS}, is estimated by performing an OLS regression conditional on the months in which the excess market return is below its average across the estimation period. In accordance with Ang et al. (2006a), we also calculate the downside beta spread, defined as $\beta_j^{DS} - \beta_j$. Following Harvey and Siddique (2000), we calculate coskewness as

$$\beta_j^{SKD} = \frac{E[\epsilon_j (R_m^e - \bar{R}_m^e)^2]}{\sqrt{E[\epsilon_j^2]} E[(R_m^e - \bar{R}_m^e)^2]}, \tag{5.1}$$

where $\epsilon_j = R_j^e - \alpha_j - \beta_j R_m^e$, the residual from a regression of the excess asset return on the excess market return. Following Dittmar (2002), we also control for cokurtosis, which we calculate as

$$\beta_j^{KUD} = \frac{E[\epsilon_j (R_m^e - \bar{R}_m^e)^3]}{\sqrt{E[\epsilon_j^2]} (E[(R_m^e - \bar{R}_m^e)^2])^{3/2}}. \tag{5.2}$$

For each characteristic we construct a risk factor by the difference between the value weighted return of stocks with estimates above the 70th percentile and the value weighted returns of stocks below the 30th percentile.[8]

After adding factors on downside beta or the downside beta spread to the FF3 benchmark model in Table 7, the results do not change much. During the extremely adverse months, the additional loss of the tail beta spread portfolio remains at a level of about −5%. The significance of the result decreases somewhat after controlling for downside beta spread as the t-statistic increases from −8.0 to −7.3. The robustness of the results is in line with the descriptive statistics. Within the sorts on the (spread of) tail beta, the average regular market beta is very similar to the average downside beta. Consequently, the risk-adjusted returns should not change much after adding downside beta factors.

We also extend the FF3 benchmark model with coskewness and cokurtosis factors. From the descriptive statistics the portfolios with high tail beta spreads have, on average, more negative estimates for coskewness. Adding the coskewness factor may thus poten-

[8] Following Harvey and Siddique (2000), we construct the zero investment portfolio for coskewness from a *short* position in the stocks with coskewness estimates above the 70th percentile and a *long* position in the stocks with coskewness estimates below the 30th percentile.

Table 7: Risk factors capturing the nonlinear relation with the market

Factors	$R^e_{m,t}$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
FF3	All	0.17	0.18	0.06	0.02	0.06	0.10
+ Downside beta		(1.2)	(2.2)	(1.0)	(0.3)	(0.5)	(0.5)
	Adverse	-2.76	-1.06	-0.14	0.99	2.95	-5.71
		(-6.1)	(-3.1)	(-0.6)	(3.7)	(6.5)	(-7.5)
	Usual	0.50	0.32	0.08	-0.09	-0.27	0.77
		(3.9)	(4.1)	(1.4)	(-1.6)	(-2.8)	(4.3)
FF3	All	0.16	0.24	0.10	0.02	-0.02	0.18
+ Spread ($\beta^{DS} - \beta$)		(1.1)	(3.0)	(1.6)	(0.3)	(-0.1)	(0.8)
	Adverse	-2.48	-0.75	-0.17	0.79	2.55	-5.03
		(-5.8)	(-2.1)	(-0.7)	(3.0)	(5.8)	(-7.3)
	Usual	0.46	0.35	0.13	-0.07	-0.31	0.77
		(3.5)	(4.3)	(2.3)	(-1.1)	(-3.1)	(4.1)
FF3	All	0.12	0.20	0.07	0.02	-0.02	0.14
+ Coskewness		(0.9)	(2.5)	(1.1)	(0.3)	(-0.2)	(0.6)
	Adverse	-2.72	-0.84	-0.27	0.71	2.62	-5.34
		(-6.2)	(-2.3)	(-1.1)	(2.6)	(6.1)	(-7.8)
	Usual	0.44	0.32	0.11	-0.06	-0.32	0.76
		(3.4)	(4.0)	(1.9)	(-1.0)	(-3.2)	(4.1)
FF3	All	0.14	0.23	0.10	0.03	-0.01	0.14
+ Coskewness		(0.9)	(2.8)	(1.5)	(0.5)	(-0.0)	(0.6)
+ Cokurtosis	Adverse	-2.63	-0.93	-0.29	0.73	2.71	-5.34
		(-5.7)	(-2.7)	(-1.1)	(2.8)	(6.4)	(-7.6)
	Usual	0.45	0.36	0.14	-0.05	-0.32	0.77
		(3.4)	(4.4)	(2.4)	(-0.8)	(-3.0)	(4.0)
FF3	All	0.15	0.27	0.10	0.05	0.03	0.12
+ Spread ($\beta^{DS} - \beta$)		(1.0)	(3.2)	(1.5)	(0.8)	(0.2)	(0.5)
+ Coskewness	Adverse	-2.45	-0.88	-0.25	0.78	2.68	-5.13
+ Cokurtosis		(-5.2)	(-2.5)	(-1.0)	(3.1)	(5.6)	(-6.7)
	Usual	0.45	0.40	0.14	-0.03	-0.27	0.72
		(3.4)	(4.8)	(2.3)	(-0.5)	(-2.7)	(3.8)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (3.2) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 value weighted (VW) portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate risk-adjusted returns by applying (3.5) on the monthly stock return at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t. Three lines of results are reported for each robustness check. First, the average risk-adjusted return across all months; second, the average risk-adjusted return across months with an excess market return below -5%; third, the average risk-adjusted return across the months with market returns above -5%.
The first row reports results after adding the downside beta factor to the FF3 benchmark model. The second row reports the results after adding the factor based on the spread between downside beta and the regular market beta to the FF3 benchmark model. In the third row we include the coskewness factor in the benchmark model. The fourth row reports the results after including the coskewness factor and cokurtosis factor to the FF3 benchmark model. The last row reports the results after adding the coskewness, cokurtosis and the downside beta spread factors to the FF3 benchmark model. The numbers in parentheses are Newey-West corrected t-statistics for the average returns across all months, and standard t-statistics for conditional averages.

tially weaken our results. However, adding coskewness and cokurtosis does not alter our results much. If both factors are added, the additional loss of the tail beta spread portfolio is estimated at −5.34% with a t-statistic of −7.6 during extremely adverse months, while the premium during usual market conditions is 0.77% with a t-statistic of 4.0.

We further consider several factors related to time dynamics in stock returns. Jegadeesh and Titman (1993) report a persistence in the returns based on the performance during the past 3 to 12 months. Based on this result, Carhart (1997) extends the FF3 model by including a momentum factor. Further, De Bondt and Thaler (1985) find a long term reversal in stock returns based on the performance over the past 3 to 5 years, while Jegadeesh (1990) reports a short-term reversal based on the performance over the last month.

To test whether these findings explain part of our results, we add the momentum factor to the benchmark model in Table 8. The additional loss during extremely adverse months on the high spread minus low spread portfolio cannot be explained by momentum. The magnitude of the loss during adverse market conditions hardly changes and remains significant with a t-statistic of −7.7. Interestingly, if the momentum factor is added, the premium during the usual months decreases from 0.74% to 0.44%. However, the premium remains significant with a t-statistic of 2.6. The decrease in the premium received during usual market days reduces the overall premium on the zero investment portfolio from 0.14 to −0.13 (both insignificant). Adding factors for long-term and short-term reversal does not change this picture much.

Pastor and Stambaugh (2003) cite anecdotical evidence on the withdrawal of liquidity around market crashes. They observe the sharpest troughs in their liquidity measure during months with significant financial and economic events, such as the 1987 crash and the 1998 collapse of LTCM. To check whether the sensitivity to the aggregate liquidity factor can explain part of our results, we add the liquidity factor from Pastor and Stambaugh (2003) to the FF3 benchmark model. The results remain practically unchanged after this addition.

Table 8: Robustness for other risk factors

Factors	$R^e_{m,t}$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
FF3	All	0.00	0.15	0.11	0.07	0.12	-0.13
+ Momentum		(-0.0)	(2.3)	(2.0)	(1.3)	(1.0)	(-0.6)
	Adverse	-2.48	-0.57	-0.09	0.81	2.40	-4.88
		(-6.1)	(-2.0)	(-0.5)	(3.4)	(6.0)	(-7.7)
	Usual	0.29	0.24	0.14	-0.02	-0.15	0.44
		(2.4)	(3.4)	(2.6)	(-0.4)	(-1.7)	(2.6)
FF3	All	0.12	0.20	0.10	0.05	0.00	0.12
+ Reversal		(1.0)	(2.8)	(1.8)	(0.9)	(0.0)	(0.6)
	Adverse	-2.12	-0.46	-0.08	0.71	2.18	-4.30
		(-4.9)	(-1.4)	(-0.4)	(3.0)	(5.5)	(-7.0)
	Usual	0.38	0.28	0.12	-0.03	-0.26	0.64
		(3.2)	(3.7)	(2.4)	(-0.6)	(-2.8)	(3.7)
FF3	All	-0.01	0.18	0.13	0.11	0.15	-0.16
+ Momentum		(-0.1)	(2.7)	(2.3)	(2.0)	(1.2)	(-0.8)
+ Reversal	Adverse	-2.20	-0.39	-0.01	0.84	2.30	-4.50
		(-5.0)	(-1.1)	(-0.1)	(3.5)	(5.4)	(-7.2)
	Usual	0.25	0.25	0.14	0.02	-0.11	0.35
		(2.0)	(3.4)	(2.8)	(0.4)	(-1.1)	(2.0)
FF3	All	0.18	0.21	0.09	0.00	-0.08	0.26
+ Liquidity		(1.5)	(2.8)	(1.5)	(-0.0)	(-0.7)	(1.3)
	Adverse	-2.27	-0.66	-0.16	0.78	2.30	-4.57
		(-5.0)	(-2.0)	(-0.7)	(3.1)	(5.4)	(-6.8)
	Usual	0.47	0.31	0.12	-0.09	-0.35	0.81
		(3.6)	(4.0)	(2.2)	(-1.6)	(-3.6)	(4.5)
FF3	All	0.14	0.26	0.15	0.09	0.12	0.02
+ Momentum		(1.2)	(3.8)	(2.5)	(1.5)	(0.9)	(0.1)
+ Reversal	Adverse	-1.81	-0.29	0.09	0.86	2.26	-4.07
+ Liquidity		(-3.8)	(-0.8)	(0.4)	(3.2)	(4.7)	(-6.2)
	Usual	0.36	0.32	0.16	0.00	-0.13	0.49
		(2.9)	(4.1)	(2.7)	(0.0)	(-1.3)	(2.7)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (3.2) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate risk-adjusted returns by applying (3.5) on the monthly stock return at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t. Three lines of results are reported for each robustness check. First, the average risk-adjusted return across all months; second, the average risk-adjusted return across months with an excess market return below -5%; third, the average risk-adjusted return across the months with market returns above -5%.

The first row reports results after adding the momentum factor to the FF3 benchmark model. The second row reports the results after adding the long-term en short-term reversal factor to the FF3 benchmark model. In the third row both reversal factors and the momentum factor are included in the benchmark model. The fourth row reports the results after including the liquidity factor to the FF3 benchmark model. The last row reports the results after including all these factors to the FF3 benchmark model. The numbers in parentheses are Newey-West corrected t-statistics for the average returns across all months, and standard t-statistics for conditional averages.

5.2 Stock characteristics

To provide further evidence that our results are not due to stock characteristics previously documented in the literature, we provide results after presorting on several characteristics. That is, at $t-1$ we first presort firms based on a certain characteristic, and then we sort on tail beta within each cohort. We report the results of this procedure with size as presorting variable in Table 3. In this subsection we focus on other characteristics. In Table 9 we report the results after averaging within each tail beta quintile across the different cohorts based on the presorting characteristic.

Table 9 shows that the results are qualitatively not affected by presorting on downside beta, coskewness and cokurtosis. To capture short-term reversal, momentum and long-term reversal, we presort stocks on the return accumulated over the past month, the past 2-12 months and the past 13-60 months. The baseline results do not change much after presorting on those characteristics.

Gervais et al. (2001) document a high-volume premium. In Table 1 we observe that firms with a high spread between tail beta and market beta have a trading volume that is on average twice as low as firms with a low spread. To test whether our findings are related to trading volume, we presort the firms on trading volume. Also after controlling for trading volume the baseline results are unaffected.

5.2.1 Idiosyncratic volatility

We provide results on idiosyncratic volatility in more detail, because we observe a pattern after presorting on this characteristic. Following Ang et al. (2006b) we concentrate on idiosyncratic volatility relative to the FF3 benchmark model, which is measured by the standard deviation of the residuals obtained from regressing the individual stock returns on the FF3 factors. Table 1 reports a U-shaped relation between idiosyncratic volatility and the size of the spread between tail beta and market beta. The observed U-shape is intuitive. Both, a relatively high (positive) and a relatively low (negative) spread between tail beta and market beta, indicate a deviation from a linear relation with the market. If idiosyncratic risk is measured by the standard deviation of the residuals obtained from

Table 9: Results after presorting

Presorting	$R^e_{m,t}$	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
Spread ($\beta^{DS} - \beta$)	All	0.13	0.17	0.08	-0.06	-0.06	0.19
		(1.1)	(2.4)	(1.4)	(-1.1)	(-0.5)	(1.0)
	Adverse	-2.24	-0.65	0.03	0.59	2.47	-4.71
		(-6.1)	(-2.1)	(0.2)	(2.8)	(6.3)	(-7.2)
	Usual	0.42	0.27	0.08	-0.13	-0.36	0.77
		(3.8)	(3.9)	(1.5)	(-2.5)	(-4.2)	(4.8)
Coskewness	All	0.10	0.16	0.11	-0.01	-0.05	0.14
		(0.8)	(2.3)	(2.4)	(-0.2)	(-0.4)	(0.7)
	Adverse	-2.16	-0.66	-0.14	0.64	2.49	-4.65
		(-5.6)	(-2.4)	(-0.6)	(3.0)	(7.4)	(-7.6)
	Usual	0.36	0.25	0.14	-0.09	-0.35	0.71
		(3.3)	(3.7)	(2.6)	(-1.7)	(-4.1)	(4.4)
Cokurtosis	All	0.13	0.15	0.11	-0.02	-0.06	0.19
		(1.1)	(2.2)	(2.2)	(-0.3)	(-0.6)	(1.0)
	Adverse	-2.40	-0.63	-0.09	0.66	2.47	-4.87
		(-5.8)	(-2.4)	(-0.5)	(3.0)	(7.1)	(-7.7)
	Usual	0.43	0.24	0.13	-0.10	-0.36	0.79
		(3.8)	(3.6)	(2.6)	(-1.9)	(-4.4)	(4.8)
Past 1 month	All	0.14	0.10	0.05	0.05	0.03	0.11
performance		(1.2)	(1.5)	(0.8)	(0.8)	(0.3)	(0.6)
	Adverse	-2.38	-0.80	0.15	1.02	2.55	-4.94
		(-6.5)	(-2.9)	(0.7)	(3.9)	(7.8)	(-8.9)
	Usual	0.44	0.21	0.03	-0.07	-0.27	0.70
		(3.7)	(3.2)	(0.6)	(-1.3)	(-3.1)	(4.2)
Past 2-12 months	All	0.07	0.13	-0.03	-0.03	-0.09	0.15
performance		(0.6)	(2.0)	(-0.6)	(-0.5)	(-0.9)	(0.8)
	Adverse	-2.53	-0.55	-0.32	0.79	2.60	-5.12
		(-6.7)	(-2.3)	(-1.4)	(4.1)	(7.8)	(-8.7)
	Usual	0.37	0.21	0.00	-0.13	-0.41	0.78
		(3.4)	(3.1)	(0.0)	(-2.3)	(-4.8)	(4.9)
Past 13-60 months	All	0.10	0.19	0.07	0.02	0.01	0.08
performance		(0.9)	(2.8)	(1.2)	(0.4)	(0.1)	(0.4)
	Adverse	-2.44	-0.62	0.22	0.91	2.79	-5.23
		(-6.9)	(-2.4)	(1.0)	(4.3)	(9.5)	(-10.1)
	Usual	0.40	0.28	0.05	-0.08	-0.31	0.71
		(3.5)	(4.1)	(0.9)	(-1.4)	(-3.5)	(4.4)
Volume	All	0.09	0.11	0.03	0.00	-0.09	0.18
		(0.9)	(1.9)	(0.7)	(-0.1)	(-1.0)	(1.1)
	Adverse	-2.20	-0.77	0.04	0.74	2.43	-4.63
		(-7.1)	(-3.7)	(0.2)	(3.6)	(8.2)	(-8.6)
	Usual	0.36	0.22	0.03	-0.09	-0.39	0.75
		(4.0)	(3.5)	(0.7)	(-1.8)	(-5.3)	(5.7)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (3.2) on daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We first presort the equities into five cohorts according to the stock characteristic specified in the first column. Within each cohort we form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate FF3-adjusted portfolio returns at time t by applying (3.5) on monthly stock returns, where the loadings on the risk factors are estimated by an OLS regression on monthly returns from the 60 months prior to t. The reported numbers are the risk-adjusted returns averaged within each tail beta quintile across the different cohorts based on the presorting characteristic.
In the first three rows, we report results after presorting on downside beta spread, coskewness and cokurtosis. The fourth, fifth, sixth and seventh row report results after presorting on the past 1 month return, the past 2-12 months return, the past 13-60 months return, and on trading volume in month $t-1$. The numbers in parentheses are Newey-West corrected t-statistics for the average returns across all months, and standard t-statistics for conditional averages.

a linear regression, then a larger deviation from the linear model will induce a higher perceived level of idiosyncratic risk. Therefore, everything else being equal, we expect to observe a higher level of idiosyncratic volatility in case of a larger deviation of the tail beta from the market beta. According to this intuition, the level of idiosyncratic volatility may produce a signal of the absolute spread between tail beta and market beta without revealing the sign of the spread. Therefore, in the higher idiosyncratic volatility cohort we expect larger differences between the high and low tail beta firms. This should be reflected by both higher losses on the zero investment portfolio during extremely adverse market conditions and a larger premium during usual market conditions. If this is the case, we would expect that the magnitude of our findings increases with respect to the level of idiosyncratic volatility.

To test this conjecture we provide detailed results of presorting firms on idiosyncratic volatility in Table 10. The results confirm our expectation. The difference in losses between high and low tail beta spread stocks during adverse market conditions for firms in the lowest idiosyncratic volatility quintile is 2.95%, while the difference for stocks in the highest idiosyncratic volatility quintile is 9.21%. It is further notable that the difference increases monotonically and is very significant in all idiosyncratic volatility quintiles. The same relation is observed in the premium during usual market days, although the relation is not entirely monotonic. The difference in return between high and low tail beta spread stocks during usual months for firms in the lowest idiosyncratic volatility quintile is 0.73%, while the difference for stocks in the highest idiosyncratic volatility quintile is 1.63%. To summarize, the results confirm that idiosyncratic volatility provides a signal of the magnitude of the spread between tail beta and market beta without revealing its sign.

5.3 Methodological changes

Table 11 provides the results from robustness checks for several methodological deviations. One potential reason why we do not find a positive premium on systematic tail risk is the market turmoil from 2007 until 2010. The recent crisis may have erased the

Table 10: Presorting on idiosyncratic volatility

$R^e_{m,t}$		Idiosyncratic Volatility	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
All	$\bar{R}^*_{FF3,p}$	Low	0.21	0.06	0.08	0.01	-0.13	0.34
		2	0.29	0.16	0.11	-0.19	-0.08	0.36
		3	0.18	0.13	0.17	-0.10	0.06	0.12
		4	0.16	-0.18	0.21	0.09	0.02	0.14
		High	0.01	-0.17	-0.12	-0.31	-0.48	0.48
		Avg	0.17	0.00	0.09	-0.10	-0.12	0.29
	t-stat	Low	(2.4)	(0.7)	(1.1)	(0.2)	(-1.4)	(2.4)
		2	(2.2)	(1.5)	(1.0)	(-1.8)	(-0.6)	(1.8)
		3	(1.0)	(0.9)	(1.2)	(-0.8)	(0.3)	(0.5)
		4	(0.8)	(-1.1)	(1.5)	(0.6)	(0.1)	(0.4)
		High	(0.0)	(-0.9)	(-0.6)	(-1.5)	(-1.9)	(1.2)
		Avg	(1.4)	(-0.0)	(1.3)	(-1.2)	(-0.9)	(1.4)
Adverse	$\bar{R}^*_{FF3,p}$	Low	-1.17	-0.01	0.03	0.43	1.78	-2.95
		2	-1.49	-0.07	0.24	0.22	2.08	-3.57
		3	-1.81	-0.65	0.56	1.71	2.92	-4.73
		4	-2.05	-1.36	0.68	1.50	4.06	-6.11
		High	-4.16	-1.32	0.41	1.90	5.05	-9.21
		Avg	-2.14	-0.68	0.39	1.15	3.18	-5.31
	t-stat	Low	(-3.0)	(-0.0)	(0.1)	(1.5)	(5.0)	(-6.0)
		2	(-3.7)	(-0.2)	(0.6)	(0.7)	(4.0)	(-4.9)
		3	(-3.4)	(-1.2)	(1.3)	(3.2)	(4.8)	(-5.9)
		4	(-3.1)	(-2.1)	(1.3)	(2.2)	(5.9)	(-5.9)
		High	(-4.3)	(-1.7)	(0.7)	(2.4)	(5.6)	(-6.6)
		Avg	(-4.9)	(-2.0)	(1.5)	(3.1)	(7.2)	(-7.4)
Usual	$\bar{R}^*_{FF3,p}$	Low	0.38	0.06	0.08	-0.04	-0.35	0.73
		2	0.50	0.18	0.09	-0.24	-0.34	0.83
		3	0.41	0.23	0.12	-0.32	-0.28	0.70
		4	0.42	-0.04	0.16	-0.07	-0.46	0.88
		High	0.50	-0.04	-0.18	-0.57	-1.13	1.63
		Avg	0.44	0.08	0.05	-0.25	-0.51	0.95
	t-stat	Small	(4.3)	(0.8)	(1.1)	(-0.4)	(-3.8)	(5.2)
		2	(4.1)	(1.9)	(1.0)	(-2.3)	(-2.7)	(4.3)
		3	(2.4)	(1.6)	(0.9)	(-2.2)	(-2.0)	(3.0)
		4	(2.1)	(-0.3)	(1.0)	(-0.5)	(-2.8)	(3.2)
		Large	(1.9)	(-0.2)	(-1.0)	(-2.9)	(-4.5)	(4.4)
		Avg	(4.0)	(1.0)	(0.8)	(-3.0)	(-5.0)	(5.5)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (3.2) on daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We first presort the equities into five cohorts based on the level of idiosyncratic volatility. Idiosyncratic volatility is calculated as the standard deviation of the residuals obtained from regressing monthly stock returns from the 60 months prior to t on the FF3 factors. Within each idiosyncratic volatility cohort we form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate FF3-adjusted portfolio returns at time t by applying (3.5) on monthly stock returns, where the loadings on the risk factors are estimated by an OLS regression on monthly returns from the 60 months prior to t. The table reports the average risk-adjusted returns of each portfolio. The numbers in parentheses are Newey-West corrected t-statistics for the averages across all months, and standard t-statistics for conditional averages.

potential positive premium on systematic tail risk. To test this hypothesis we repeat the tests on a subsample until 2007. The average value weighted FF3-adjusted return on the zero investment portfolio increases from 0.14% to 0.19%, but remains insignificantly different from zero with a t-stat of 1.0. Further, the positive premium on systematic tail risk during usual market days and the additional loss during extremely adverse periods remain strongly significant.

A structural break in the CRSP data is the entrance of NASDAQ firms in January 1973. After collecting five years of return data to estimate tail betas, the first NASDAQ firms enter the constructed portfolios in January 1978. To test whether the results are robust for this structural break, we repeat the asset pricing tests on portfolio returns from January 1978 onwards. The results are robust for restricting the sample horizon. Alternatively, we also repeat the asset pricing tests while restricting the sample to NYSE firms only. Although the loss during adverse months for the high minus low tail beta spread decreases from 4.90% to 4.48%, its t-statistic increases from −8.0 to −8.7. These results suggest that our findings are not sample specific.

The next robustness check is on the choice in the definition of extremely adverse market conditions. In the baseline result, we define months with an excess market return below −5% as 'extremely adverse market conditions' and those with a return above −5% as 'usual market conditions'. We test whether our results are robust for this specific choice. In the table we repeat our procedure, but include only the 20 worst months in the sample of extremely adverse months. Our results are robust for this alternative definition. The risk-adjusted loss on the zero investment portfolio increases from −4.9% to −6.5% during these more adverse months. The inclusion of potentially adverse months in the sample with usual months also shrinks the premium received during the other months, both in terms of magnitude and significance level. Nevertheless, the reduced premium remains significant.

The estimation of each tail beta is based on the 50 worst market days during the past 60 months. To test whether our results are robust for alternative choice, we also estimate the tail beta based on the 30 worst market days during the past 60 months, i.e., we set

Table 11: Methodological robustness

Robustness	$R^e_{m,t}$	Obs	High $\beta^T - \beta$	4	3	2	Low $\beta^T - \beta$	(5) - (1)
Pre-crisis	All	463	0.12	0.17	0.07	-0.01	-0.07	0.19
			(0.9)	(2.3)	(1.3)	(-0.3)	(-0.7)	(1.0)
	Adverse	44	-2.33	-0.72	-0.18	0.54	1.92	-4.25
			(-5.0)	(-2.2)	(-0.8)	(2.1)	(4.9)	(-6.5)
	Usual	419	0.38	0.26	0.10	-0.07	-0.29	0.66
			(3.0)	(3.4)	(1.9)	(-1.3)	(-3.3)	(3.8)
Post NASDAQ Break	All	397	0.14	0.17	0.12	-0.01	-0.01	0.15
			(1.0)	(2.0)	(1.9)	(-0.1)	(-0.1)	(0.6)
	Adverse	40	-2.59	-0.83	-0.07	0.79	2.93	-5.51
			(-5.4)	(-2.2)	(-0.3)	(2.6)	(6.5)	(-7.4)
	Usual	357	0.45	0.29	0.14	-0.10	-0.34	0.79
			(3.1)	(3.3)	(2.3)	(-1.5)	(-3.3)	(3.9)
NYSE stocks only	All	510	0.11	0.15	0.03	-0.03	-0.13	0.23
			(1.0)	(2.6)	(0.6)	(-0.5)	(-1.2)	(1.4)
	Adverse	54	-1.66	-0.32	0.29	0.68	2.43	-4.09
			(-4.1)	(-1.4)	(1.2)	(2.6)	(6.8)	(-7.3)
	Usual	456	0.31	0.21	0.00	-0.12	-0.43	0.75
			(3.2)	(3.7)	(0.1)	(-2.0)	(-4.7)	(4.7)
More adverse months	All	510	0.11	0.17	0.08	0.02	-0.03	0.14
			(0.9)	(2.4)	(1.4)	(0.4)	(-0.3)	(0.7)
	Adverse	20	-3.36	-0.92	-0.32	0.90	3.13	-6.49
			(-5.0)	(-1.9)	(-1.0)	(2.1)	(4.2)	(-5.4)
	Usual	490	0.25	0.22	0.10	-0.02	-0.16	0.41
			(2.1)	(3.0)	(1.8)	(-0.3)	(-1.8)	(2.3)
Lower tail threshold	All	510	0.02	0.17	0.09	-0.01	-0.02	0.04
			(0.2)	(2.5)	(1.7)	(-0.2)	(-0.2)	(0.2)
	Adverse	54	-2.28	-0.57	-0.09	0.63	2.48	-4.77
			(-5.6)	(-2.1)	(-0.4)	(3.0)	(5.3)	(-7.0)
	Usual	456	0.29	0.26	0.11	-0.09	-0.32	0.61
			(2.5)	(3.6)	(2.1)	(-1.5)	(-3.4)	(3.5)
Conditional approach	All	510	0.08	0.13	0.03	-0.13	-0.06	0.14
			(1.1)	(2.6)	(0.5)	(-1.6)	(-0.4)	(0.7)
	Adverse	54	-1.23	0.26	0.78	1.09	3.61	-4.84
			(-4.1)	(1.4)	(3.4)	(4.0)	(6.2)	(-7.3)
	Usual	456	0.24	0.11	-0.06	-0.27	-0.50	0.73
			(3.5)	(2.1)	(-0.9)	(-3.4)	(-4.0)	(4.4)

Note: At the start of each month t between July 1968 and Dec 2010, we estimate tail betas for NYSE, AMEX and NASDAQ equities by applying the EVT approach in (3.2) on past daily returns from the 60 months prior to t. Market betas are estimated based on monthly returns from the same horizon. We form 5 value weighted portfolios by sorting on the spread between tail beta and market beta and construct a zero-investment portfolio. We calculate FF3-adjusted returns by applying (3.5) on the monthly stock return at time t, where the loadings on the risk factors in the benchmark model are estimated for each stock by an OLS regression on monthly returns from the 60 months prior to t. Three lines of results are reported for each robustness check. First, the average FF3-adjusted return across all months; second, the average FF3-adjusted return across months with an excess market return below a certain threshold; third, the average FF3-adjusted return across the months with market returns above this threshold. If the threshold is not specified, then it is fixed at -5%.
The first row reports results based on the pre-crisis sample: from July 1968 to Dec 2006. The second row reports results based on the sample after the entrance of NASDAQ firms: from Jan 1978 to Dec 2010. The third row varies the threshold such that only 20 months are selected in the sample with extremely adverse market conditions (the corresponding threshold level is -8.1%). The fourth row reports the results if tail betas are estimated with $k = 30$ instead of $k = 50$ in (3.2). The fifth row reports the result based on estimating the tail beta by a conditional regression approach, based on the observations corresponding to the 50 worst daily market excess returns during the months between $t - 60$ and $t - 1$. The numbers in parentheses are Newey-West corrected t-statistics for the averages across all months, and standard t-statistics for conditional averages.

$k = 30$ and estimate tail beta based on about $k/n \approx 2.5\%$ of the worst market days. The results do barely change, although the t-statistic of the premium during usual market days decreases somewhat from 4.3 to 3.5.

Finally, we test whether the results are robust for using an alternative estimator for tail beta. Instead of using the EVT estimator in (3.2), we apply the conditional regression approach on daily returns. That is, we perform an OLS regression based on the 50 days with the worst daily market returns in each estimation window. The results remain practically unchanged, both in terms of magnitude and significance.

To summarize, the robustness checks suggest that our baseline results are robust for several methodological changes, and for controlling several firm characteristics that have been documented in the literature to explain the cross-section of returns. Two findings are notable. First, adding the momentum factor reduces the premium that portfolios with high tail betas receive during usual periods, without reducing the additional losses that these portfolios suffer during extremely adverse months. Second, we find evidence that idiosyncratic risk provides a signal about the magnitude of the deviation between tail beta and market beta.

6 Risk management

Since historical tail betas can capture future losses under extremely adverse market conditions, tail betas may help investors assess the tail risk of portfolios. As an additive measure of loading on systematic tail risk, the tail beta is also a useful measure in the context of managing the tail risks of portfolios. We discuss this application in the current section.

We consider a portfolio consisting of d assets, following the linear tail model in (2.1) with nonnegative tail betas, $\beta_1^T, \cdots, \beta_d^T$. Then, under extremely adverse market conditions, the excess return of a portfolio with non-negative investment weights, $w_1, \cdots, w_d$, can be written as

$$R_P^e = \left(\sum_{j=1}^{d} w_j \beta_j^T\right) R_m^e + \sum_{j=1}^{d} w_j \varepsilon_j, \quad \text{for} \quad R_m^e < -VaR_m(\bar{p}). \tag{6.1}$$

Hence, the portfolio return also follows a linear tail model with a portfolio tail beta equal to the weighted average of the tail beta of the individual assets, i.e., $\beta_P^T = \sum_{j=1}^{d} w_j \beta_j^T$ and an idiosyncratic component that is given by $\varepsilon_P = \sum_{j=1}^{d} w_j \varepsilon_j$.

To evaluate the tail risk of a portfolio, it is necessary to aggregate the systematic and idiosyncratic tail risks. We start by discussing the aggregation for a single asset. Suppose the linear tail model in (2.1) and the heavy-tailed setup in (3.1) hold for a larger area, $\min(R_m^e, R_j^e) < -VaR_m(\bar{p})$. It then follows that the probability of a loss on asset j larger than u can be approximated by

$$\Pr(R_j^e < -u) \sim \Pr(\beta_j^T R_m^e < -u) + \Pr(\varepsilon_j < -u), \quad \text{as} \quad u \to \infty. \tag{6.2}$$

This approximation follows from Feller's convolution theorem on aggregating risk factors, which states that the probability that the sum of independent heavy-tailed risk factors is above a high threshold can be approximated by the sum of the probabilities of each risk factor being above that threshold; see Feller (1971, p. 278).[9] Suppose the idiosyncratic risk, ε_j, follows a heavy-tailed distribution with tail index α_{ε_j} and scale A_{ε_j}.[10] If $\alpha_{\varepsilon_j} > \alpha_m$, then the systematic tail risk dominates the idiosyncratic tail risk, i.e., $\Pr(\varepsilon_j < -u) = o(\Pr(\beta_j^T R_m^e < -u))$ as $u \to \infty$. Consequently, the downside tail distribution of the excess asset return, R_j^e, follows a heavy-tailed distribution with tail index $\alpha_j = \alpha_m$ and scale $A_j = (\beta_j^T)^{\alpha_m} A_m$. In contrast, if $\alpha_{\varepsilon_j} < \alpha_m$, then the idiosyncratic risk dominates the tail risk of the asset, and we have $\alpha_j = \alpha_{\varepsilon_j}$ and $A_j = A_{\varepsilon_j}$. In the case $\alpha_{\varepsilon_j} = \alpha_m$, both of the two components contribute to the tail risk of the asset, and we have $A_j = (\beta_j^T)^{\alpha_m} A_m + A_{\varepsilon_j}$.

In the portfolio context, we first consider the case $\alpha_{\varepsilon_1} = \cdots = \alpha_{\varepsilon_d} = \alpha_m$. Suppose the assets have independent idiosyncratic tail risks with scales $A_{\varepsilon_1}, \cdots, A_{\varepsilon_d}$. Following

[9]Embrechts et al. (1997), Lemma 1.3.1., provides the proof for the case $\alpha_m = \alpha_{\varepsilon_j}$. Along the same lines of proof one can obtain that this relation holds for $\alpha_m \neq \alpha_{\varepsilon_j}$.

[10]A thin-tailed idiosyncratic risk could be thought of as having $\alpha_{\varepsilon_j} = \infty$ in the following discussion.

Feller's convolution theorem, the downside tail of the portfolio follows a heavy-tailed distribution with tail index $\alpha_P = \alpha_m$ and scale

$$A_P = (\beta_P^T)^{\alpha_m} A_m + \sum_{j=1}^{d} w_j^{\alpha_m} A_{\varepsilon_j}. \tag{6.3}$$

In practice, all parameters in equation (6.3) can be statistically estimated. In particular, the tail beta of the portfolio, β_P^T, can be obtained by taking a weighted average of the tail beta estimates of the individual assets, the $\hat{\beta}_j^T$s. Furthermore, the scales of the idiosyncratic tail risks, A_{ε_j}, can be obtained from

$$\hat{A}_{\varepsilon_j} = \hat{A}_j - (\hat{\beta}_j^T)^{\hat{\alpha}_m} \hat{A}_m, \tag{6.4}$$

where the scales of the market return and the asset return, A_m and A_j, can be estimated by univariate EVT analysis; see e.g. Hill (1975). With equation (6.3), we thus obtain the estimate of the scale of a portfolio. Subsequently, the VaR of the portfolio for some low probability level p can be calculated from the approximation

$$VaR_P(p) \approx \left(\frac{A_P}{p} \right)^{1/\alpha_m}. \tag{6.5}$$

Next, consider the case in which some assets in the portfolio correspond to $\alpha_{\varepsilon_j} > \alpha_m$. The idiosyncratic tail risks of those assets are dominated by their systematic tail risk and do not contribute to the tail risk of the portfolio. Hence, it is still possible to evaluate the scale of the portfolio with equation (6.3) by omitting the idiosyncratic tail risks of those assets. However, it is not necessary to identify those assets or to modify the estimation procedure from equations (6.3) and (6.4). Assets with $\alpha_{\varepsilon_j} > \alpha_m$ exhibit complete tail dependence with the market return, i.e., $\tau_j = 1$ and $A_j = (\beta_j^T)^{\alpha_m} A_m$. Therefore, the estimator on A_{ε_j} in (6.4) converges to zero under the EVT approach. Including the estimate of A_{ε_j} for such assets in equation (6.3) will not contaminate the estimate of the portfolio scale. In summary, equation (6.3) can be applied to any portfolio consisting of assets with $\alpha_{\varepsilon_j} \geq \alpha_m$.

Finally, we discuss the case in which some assets correspond to $\alpha_{\varepsilon_j} < \alpha_m$. Theoretically, the downside tail risk of the portfolio would be dominated by the idiosyncratic risk of the asset with the lowest tail index. However, in practice this may not be the case. The reason is that the return on many assets is in fact bounded from below by -100%.[11] Such an asset j with investment weight w_j can generate a maximum loss of w_j. Therefore, in a well-diversified portfolio with a sufficiently large number of assets, the idiosyncratic tail risks do not contribute to the tail risk of the portfolio under the condition that their returns have a lower bound. This is achieved even if some assets correspond to the case $\alpha_{\varepsilon_j} < \alpha_m$.[12] In contrast to the idiosyncratic risks, the systematic tail risk cannot be diversified away by investing in a large number of assets, because the tail beta of a portfolio is the weighted average of those of the individual assets. Hence, for any well-diversified portfolio consisting of a sufficiently large number of assets with lower bounded returns, the scale of its downside tail distribution can be approximated by

$$A_P = \left(\sum_{j=1}^{d} w_j \beta_j^T \right)^{\alpha_m} A_m.$$

Subsequently, the VaR can be calculated from equation (6.5).

7 Concluding remarks

This paper investigates whether systematic tail risk is compensated in the cross-section of expected returns. Asset pricing theory based on an equilibrium framework with safety-first investors suggests that higher loadings on systematic tail risk should be associated with a positive risk premium. Theoretically, the risk premium for any asset is proportional to its tail beta, which measures the sensitivity to systematic tail risk. Based on an EVT approach, we estimate tail betas and test empirically whether stocks with

[11]Examples of assets of which the returns have a lower bound are long positions in stocks and bonds. Counterexamples are short positions in currencies and stocks.

[12]The lower bound of equity returns is not accounted for in the heavy tail approximation, as in (3.1). Instead, one could consider truncated heavy-tailed distributions. Ibragimov and Walden (2007) prove the diversification effects of bounded risk factors from truncated heavy-tailed distributions provided that the number of risk factors is sufficiently large.

high tail betas received higher average returns.

We find that assets with higher tail betas are associated with significantly larger losses during future extreme market downturns. Hence, historical tail betas are able to capture the sensitivity to future systematic tail risk. Further, the asset pricing tests do not report a significant positive premium for high tail beta stocks over the entire historical sample. One potential interpretation of this result is that there are measurement issues, such as time variation in the actual tail betas. Even if systematic tail risk is priced in the cross-section of expected returns, time-varying tail betas might weaken the observed premium when sorting on historical estimates. However, our historical estimates perform well in differentiating future losses under extremely adverse market conditions. Hence, such an interpretation is satisfactory only if the risk premium for loading on systematic tail risk is rather low, which suggests that the room for a positive systematic tail risk premium in the cross-section of expected returns is limited. In addition, we find weak evidence of a significant negative premium among small and medium firms, which further supports the absence of a positive premium for high tail beta stocks. Possible explanations for these results are that fund managers are less concerned with the performance of their portfolios under extremely adverse conditions than their clients, or that investors are insufficiently aware of the cross-sectional differences in systematic tail risk.

Parallel to the tail beta which measures assets' sensitivity to the extreme downside risk of the market, individual assets may also exhibit differences in their comovement with large market booms. The methodology to estimate 'downside' tail betas can also be applied to estimate 'upside' tail betas. In the same vein as the discussion on (downside) risk management, such upside tail betas may provide information on portfolio profits in a hypothetical large boom, where the upside tail beta of a portfolio is a weighted average of the upside tail betas of the individual assets. In the safety-first framework, which focuses on downside risk only, upside tail betas are irrelevant for the cross-section of expected returns. The relevance of upside tail betas in other asset pricing frameworks is left for future research.

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Appendix A: Proof of equality in equation (2.3)

We start by introducing the notation of Arzac and Bawa (1977). Let the initial and future market value of asset j be denoted by V_j and X_j. Then each asset j generates a return $R_j = X_j/V_j$. The value-weighted market portfolio has the initial and future value $V_m = \sum_j V_j$ and $X_m = \sum_j X_j$, and therefore the market return is defined as $R_m = \frac{\sum_j X_j}{\sum_j V_j} = \sum_j w_j^* R_j$, with weights $w_j^* = V_j/(\sum_j V_j)$.

Investor i holds a portfolio with fractions of the risky assets $(\gamma_{i,1}, \gamma_{i,2}, \cdots)$, which generates a future value as $\sum_j \gamma_{i,j} X_j = \sum_j \gamma_{i,j} V_j R_j$. Let us denote the p-quantile of the future value of investor i and the market portfolio as Q^i and Q_m, respectively. Then the p-quantile of the market return is $q_m = Q_m/V_m$.

With this notation, Arzac and Bawa (1977, eq. 14) give the formula to calculate the parameter β_j^{AB} as

$$\beta_j^{AB} = \frac{q_j - r_f}{q_m - r_f}.$$

Here q_j is given by

$$q_j := \frac{\frac{\partial Q^i}{\partial \gamma_{i,j}} \,|_{(\gamma_{i,j})=(\gamma_i)}}{V_j},$$

where (γ_i) is the optimal portfolio holding for investor i on all assets. The right-hand side is the same across all investors.

Because $q_m - r_f$ is the $p-$quantile of the market excess return, we have that $q_m - r_f = -VaR_m(p)$. Therefore, to prove the equality in equation (2.3), it is only necessary to prove that $q_j = E(R_j|R_m = Q_m(p))$, where $Q_m(p) = q_m$ is the $p-$quantile of the return of the market portfolio.

To relate the quantile of the future value of investors' portfolio to that of the market return, we define for any positive investments $(u_1, u_2, \cdots)$ the $p-$quantile of $\sum_j u_j R_j$ as $f(u_1, u_2, \cdots)$. Notice that $Q_m = f(V_1, V_2, \cdots)$, $Q^I = f(\gamma_{i,1}V_1, \gamma_{i,2}V_2, \cdots)$. We calculate q_j as

$$q_j = \frac{\frac{\partial f(\gamma_{i,1}V_1, \gamma_{i,2}V_2, \cdots)}{\partial \gamma_{i,j}} \,|_{(\gamma_{i,j})=(\gamma_i)}}{V_j} = \frac{V_j \frac{\partial f}{\partial u_j} \,|_{(u_j)=(\gamma_i V_j)}}{V_j} = \frac{\partial f}{\partial u_j} \,|_{(u_j)=(\gamma_i V_j)} .$$

The function f is homogeneous with degree one, which implies that its partial derivative

$\frac{\partial f}{\partial u_j}$ is a homogeneous function with degree zero. Consequently, we have

$$\frac{\partial f}{\partial u_j} |_{(u_j)=(\gamma_i V_j)} = \frac{\partial f}{\partial u_j} |_{(u_j)=(w_j^*)} .$$

To derive the partial derivative of the f function, we use the expression that

$$f(u_1, u_2, \cdots) = E(\sum_j u_j R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,)).$$

Thus,

$$\begin{aligned}
&\frac{\partial f}{\partial u_j} |_{(u_j)=(w_j^*)} \\
=&\frac{\partial}{\partial u_j} E(\sum_j u_j R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,)) |_{(u_j)=(w_j^*)} \\
=&\frac{\partial}{\partial u_j} \sum_j u_j E(R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,)) |_{(u_j)=(w_j^*)} \\
=&E(R_j | \sum_j u_j R_j = f(u_1, u_2, \cdots,)) |_{(u_j)=(w_j^*)} \\
=&E(R_j | R_m = Q_m(p)).
\end{aligned}$$

The last equality follows from the fact that $R_m = \sum_j w_j^* R_j$ and $Q_m(p) = f(w_1^*, w_2^*, \cdots)$. The q_j thus quantifies the contribution of asset j to the $p-$quantile of the market return.

□

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