Optimal Hedge Fund Allocation with Improved Estimates for Coskewness and Cokurtosis Parameters

September 2010
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The present document, which represents the second-year publication from the “Advanced Modelling for Alternative Investments” research chair at EDHEC-Risk Institute, supported by Newedge Prime Brokerage, is a perfect illustration of the purpose behind the research chair, namely to expand the frontiers in alternative investment modelling techniques by enhancing the understanding of the dynamic and non-linear relationship between alternative investment returns and the returns on underlying fundamental systematic factors, and analysing the implications for managing portfolios that include alternative investments.

It is clear that since hedge fund returns are not normally distributed, mean-variance optimisation techniques, which would lead to substantial welfare losses from the investor’s perspective, need to be replaced by optimisation procedures incorporating higher-order moments and comoments. In this context, optimal portfolio decisions involving hedge fund style allocation require not only estimates for covariance parameters but also estimates for coskewness and cokurtosis parameters.

The current publication presents an application of the improved estimators for higher-order comoment parameters, recently introduced by Martellini and Ziemann (2010), in the context of hedge fund portfolio optimisation. The authors find that the use of these enhanced estimates generates significant improvement for investors in hedge funds, and that it is only when improved estimators are used that portfolio selection with higher-order moments consistently dominates mean-variance analysis from an out-of-sample perspective.

As the authors point out, the results have important potential implications for hedge fund investors and hedge fund of funds managers who routinely use portfolio optimisation procedures incorporating higher moments.

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Wishing you an agreeable and informative read,

Noël Amenc
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Abstract
Abstract

Since hedge fund returns are not normally distributed, mean-variance optimisation techniques, which would lead to substantial welfare losses from the investor’s perspective, need to be replaced by optimisation procedures incorporating higher-order moments and comoments. In this context, optimal portfolio decisions involving hedge fund style allocation require not only estimates for covariance parameters but also estimates for coskewness and cokurtosis parameters.

This is a formidable challenge that severely exacerbates the dimensionality problem already present with mean-variance analysis. This paper presents an application of the improved estimators for higher-order co-moment parameters, recently introduced by Martellini and Ziemann (2010), in the context of hedge fund portfolio optimisation. We find that the use of these enhanced estimates generates a significant improvement for investors in hedge funds. We also find that it is only when improved estimators are used that portfolio selection with higher-order moments consistently dominates mean-variance analysis from an out-of-sample perspective. Our results have important potential implications for hedge fund investors and hedge fund of funds managers who routinely use portfolio optimisation procedures incorporating higher moments.
1. Introduction
One of the main reasons why institutional investors are willing to include hedge funds in their portfolios is related to their expected diversification benefits with respect to other existing investment possibilities. In a nutshell, the diversification argument states that investors can take advantage of hedge funds' linear and non-linear exposure to a great variety of risk factors, including volatility, credit and liquidity risk, etc., to reduce the risk of their global portfolio. That mixing hedge funds with traditional assets leads to a reduction in the volatility of the traditional portfolio with a constant return level had been underlined by many (see for example Terhaar et al. 2003), and originates from the fact that hedge funds (at least for some strategies) present low volatility together with low correlation with traditional asset classes. In an attempt to fully capitalise on diversification benefits in a top-down approach, investors or (funds of hedge funds) managers must be able to rely on robust techniques for optimisation of portfolios including hedge funds. Standard mean-variance portfolio selection techniques are known to suffer from a number of shortcomings, and the problems are exacerbated in the presence of hedge funds. First, because hedge fund returns are not normally distributed (see Brooks and Kat (2002)), a mean-variance optimization would be severely ill-adapted, except in the case of an investor endowed with quadratic preferences. For example, it can be shown, through a statistical model integrating fatter tails than those of the normal distribution, that minimising the second order moment (the volatility) can be accompanied by a significant increase in extreme risks (Sornette, Staub, and Singer. 2000). This is confirmed in Amin and Kat (2003), where the authors find empirical evidence that low volatility is generally obtained at the cost of lower skewness and higher kurtosis. Consequently, as stressed in Cremers, Kritzman, and Page (2005), in the presence of asymmetric and/or fat-tailed return distribution functions, the use of mean-variance analysis can lead to a significant loss of utility for investors.

As a result of the shortcomings of mean-variance optimization, many attempts have been made to better account for the specific risk features of hedge funds and to extend portfolio optimization techniques to account for the presence of fat-tailed distributions, mostly by introducing some risk objective (e.g., Value-at-Risk as in Favre and Galeano [2002], or Conditional Value-at-Risk as in De Souza and Gokcan [2004] and Agarwal and Naik [2004]), more general than volatility, integrating the presence of non-trivial higher moments in asset returns. In the presence of non-normally distributed asset returns, optimal portfolio selection techniques require not only estimates for variance-covariance parameters but also estimates for higher-order moments and comoments of the return distribution. The need to estimate coskewness and cokurtosis parameters severely increases, however, the dimensionality problem, already a serious concern in the context of covariance matrix estimation. This concern is particularly acute in the hedge fund universe, where data is scarce, with a short history and low frequency, and where a number of performance biases are present (Fung and Hsieh 1997, 2000, 2002). In this context, given the dramatic increase in dimensionality involved, one might wonder whether portfolio selection techniques that rely on higher-order moments can efficiently be implemented.
1. Introduction

at all in realistic situations. In a recent paper, Martellini and Ziemann (2010) shed light on this question by introducing improved estimators for the coskewness and cokurtosis parameters. They extend to the skewness and kurtosis dimensions several improved estimates that had been proposed for the covariance matrix, including most notably the factor-based approach (Sharpe 1963), the constant correlation approach (Elton and Gruber 1973) and the statistical shrinkage approach (Ledoit and Wolf 2003). In an empirical analysis based on US large cap stock returns, Martellini and Ziemann (2010) subsequently find that the use of these enhanced estimates generates significant improvement in investors’ welfare.

We complement these results by providing the first application of improved estimators for higher-order comoment parameters in the context of hedge fund portfolio optimization. We find that the use of these enhanced estimates generates significant improvement for investors in hedge funds. We also find that it is only when improved estimators are used that portfolio selection with higher-order moments dominates mean-variance analysis from an out-of-sample perspective. More specifically, we construct portfolios based on various hedge fund style indices using a fourth order approximation of expected CARA utility, and shrinkage estimators to alleviate the concern over robustness of purely sample-based estimates. We find that the use of improved estimators leads to substantial increases in the investor’s utility as compared to using sample estimators. On the other hand, we find that extending the objective function to encompass higher-order moments of hedge fund return distribution can lead to destroy, as opposed to add-value, when sample-based estimators are used. The rest of the paper is organised as follows. Section 2 introduces the improved estimators for hedge fund return covariance, coskewness and cokurtosis parameters. In section 3 we present our empirical analysis, while section 4 contains some concluding thoughts.
1. Introduction
2. Improved Estimators for Hedge Fund Return Covariance, Coskewness, and Cokurtosis Parameters
2. Improved Estimators for Hedge Fund Return Covariance, Coskewness, and Cokurtosis Parameters

We consider an investor or asset manager who maximises expected utility from the terminal wealth of a portfolio invested in hedge funds. We assume that the agent is endowed with constant absolute risk aversion (CARA) preferences expressed over final wealth $W$:

$$U(W) = -e^{-\lambda W}$$

where $\lambda$ is the investor’s risk aversion coefficient. In this context, it is straightforward to show that the investors’ expected utility of final wealth can be approximated by the following fourth-order Taylor expansion (Martellini and Ziemann 2010 for details):

$$\mathbb{E}U(W) = -e^{-\lambda \mu w'}\left[1 + \frac{\lambda^2}{2} w'M_2 w' - \frac{\lambda^3}{6} w'M_3 (w' \otimes w') + \frac{\lambda^4}{24} w'M_4 (w' \otimes w' \otimes w') \right]$$

where $w$ is the vector of weights defining the allocation to various hedge fund styles, $\mu$ is the vector containing the mean return on the hedge fund style indices, and $M_2$, $M_3$, $M_4$ are the covariance matrix, coskewness matrix and cokurtosis matrix of hedge fund style return distributions, respectively. Here the symbol $\otimes$ stands for the Kronecker product.

In this framework the objective of the investor is to maximise the expected utility at the end of the period namely:

$$\max_w -e^{-\lambda \mu w'}\left[1 + \frac{\lambda^2}{2} w'M_2 w' - \frac{\lambda^3}{6} w'M_3 (w' \otimes w') + \frac{\lambda^4}{24} w'M_4 (w' \otimes w' \otimes w') \right]$$

such that $\sum_{i=1}^N w_i = 1$ and $0 \leq w_i \leq 1$

where $N$ is the number of hedge fund styles considered in the analysis, and lower and upper bounds have been imposed to the investment in each hedge fund style to prevent short positions and leverage.

As recalled in the introduction, the number of parameters involved when higher-order moments are taken into account increases exponentially with the number of constituents in the portfolio. Given the total numbers of parameters to estimate and the relatively small sample size induced by the short history and low frequency of hedge fund return data, simple sample-based estimators for higher-order comoments will likely contain a substantial amount of estimation error even for relatively small portfolios. In an attempt to address this concern, the literature on improved risk estimates, focusing first and foremost on the variance-covariance matrix, has proposed to impose some structure on the higher-order moment matrices so as to reduce the number of parameters involved. The first improved estimator that has been introduced is the constant correlation estimator, proposed by Elton and Gruber (1973). In a nutshell, they argue that imposing the assumption of a constant correlation across assets, while obviously involving a substantial amount of specification error, leads to improved out-of-sample portfolio performance. An unbiased estimator for the constant correlation parameter is given by the average over all sample correlation parameters:

$$\hat{\rho} = \frac{2}{N(N-1)} \sum_{i<j} \hat{r}_{ij} \quad \text{with} \quad \hat{r}_{ij} = \frac{s_{ij}}{\sqrt{s_{ii}s_{jj}}}$$

where $s$ denotes sample variances and covariances. Following this approach, the covariance ($\sigma_{ij}$) parameters can be
2. Improved Estimators for Hedge Fund Return Covariance, Coskewness, and Cokurtosis Parameters

estimated as a function of the constant correlation parameter and the asset volatility parameters $\sigma_{ij} = \sqrt{s_{ii}s_{jj}}$, thus allowing for a dramatic reduction in the number of parameters. Subsequently, Martellini and Ziemann (2010), derive the counterparts of correlation coefficients for higher-order comoments and extend the concept of a constant correlation to the context of higher-order comoments.¹

To do so, they introduce the following extended correlation coefficients, where $r^{(1)}$ is the standard correlation coefficient and where $\overline{R}$ denote centred returns, that is, $\overline{R} = R - \mu$:

\[ r_{ij}^{(1)} = \frac{E(R_i R_j)}{\sqrt{E(R_i^2) E(R_j^2)}} \]
\[ r_{ij}^{(2)} = \frac{E(R_i^2 R_j)}{\sqrt{E(R_i^2) E(R_j^2)}} \]
\[ r_{ij}^{(3)} = \frac{E(R_i^3 R_j)}{\sqrt{E(R_i^3) E(R_j^3)}} \]
\[ r_{ij}^{(4)} = \frac{E(R_i^4 R_j)}{\sqrt{E(R_i^4) E(R_j^4)}} \]
\[ r_{ij}^{(5)} = \frac{E(R_i^5 R_j)}{\sqrt{E(R_i^5) E(R_j^5)}} \]
\[ r_{ij}^{(6)} = \frac{E(R_i^6 R_j)}{\sqrt{E(R_i^6) E(R_j^6)}} \]
\[ r_{ij}^{(7)} = \frac{E(R_i^7 R_j)}{\sqrt{E(R_i^7) E(R_j^7)}} \]  

Consistent with Elton and Gruber (1973), and following Martellini and Ziemann (2010), we shall assume in the empirical analysis that the parameters $r^{(1)} - r^{(7)}$ are constant across hedge fund style returns. In other words, we use the following estimators, corresponding to sample average correlations across assets:

\[ \hat{r}_{ij} = \frac{1}{T} \sum_{t=1}^{T} \left( \frac{R_{it} R_{jt}}{\sqrt{\overline{R}_{it}^2 \overline{R}_{jt}^2}} \right) \]

Where $\overline{R}_{it}$ denotes the centered return of strategy $i$ at time $t$. $m^{(n)}_i$ is the $n$-th centred sample moment of strategy $i$ given by:

\[ m^{(n)}_i = \frac{1}{T} \sum_{t=1}^{T} (\overline{R}_{it}^n)^{\frac{1}{n}} \]

Replacing individual sample correlations by those seven constant coefficients then allows us to obtain all elements in $M_2$, $M_3$, and $M_4$ by merely using sample estimates for the second, third, fourth, and sixth central moments:

\[ \hat{\kappa}_{ii} = \hat{r}_{ii}^{(3)} \sqrt{m_3^{(i)} m_4^{(i)}} \]
\[ \hat{\kappa}_{ij} = \hat{r}_{ij}^{(3)} \sqrt{m_3^{(i)} m_4^{(j)}} \]
\[ \hat{\kappa}_{ik} = \hat{r}_{ik}^{(3)} \sqrt{m_3^{(i)} m_4^{(k)}} \]
\[ \hat{\kappa}_{ij} = \hat{r}_{ij}^{(4)} \sqrt{m_4^{(i)} m_5^{(j)}} \]
\[ \hat{\kappa}_{ik} = \hat{r}_{ik}^{(4)} \sqrt{m_4^{(i)} m_5^{(k)}} \]
\[ \hat{\kappa}_{jk} = \hat{r}_{jk}^{(4)} \sqrt{m_4^{(j)} m_5^{(k)}} \]

While the constant correlation estimator involves lower estimation risk due to the substantial increase in the number of parameters to be estimated, it also involves some misspecification risk inherent to the structure artificially imposed. In an attempt to find the optimal trade-off between sample risk and specification error, Ledoit and Wolf (2003) have introduced in the context of the covariance matrix the asymptotically optimal linear combination of the sample estimator and a structured estimator (constant correlation), with the weight assigned to the latter being known as optimal shrinkage intensity. In the empirical analysis of hedge fund returns, we do not use the constant correlation estimators, but instead use a mixture of that estimator with the sample-based estimator. To do so, we use the results in Martellini and Ziemann (2010), to which we refer for
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further technical details regarding how to extend the shrinkage estimators to higher-order moments by finding the ex ante optimal linear combinations of sample and constant-correlation estimators that minimises the posterior misspecification function of the combined estimator.
3. Empirical Analysis
3. Empirical Analysis

We now turn to an empirical analysis of the out-of-sample performance of the sample based and improved version of the estimators introduced in the previous section.

3.1 Description of the Data Base

The focus is on constructing a portfolio on the basis of the following thirteen hedge fund strategies: Convertible Arbitrage, CTA Global, Distressed Securities, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Global Macro, Long/Short Equity, Merger Arbitrage, Relative Value, Short Selling and Fund of Funds. We use the EDHEC Alternative Indexes as proxies for the return on the selected hedge fund strategies. Overall, the time series of hedge fund strategy returns ranges from January 1997 to August 2009, consisting of 152 monthly observations. In table 1, we report some descriptive statistics for each time series of returns on the whole period under consideration.

<table>
<thead>
<tr>
<th>Hedge Fund Strategy</th>
<th>Mean Return</th>
<th>Volatility</th>
<th>Skew</th>
<th>Kurt</th>
<th>JBTest</th>
<th>PValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convertible Arbitrage</td>
<td>7.967</td>
<td>6.944</td>
<td>-2.684</td>
<td>19.178</td>
<td>1840.10</td>
<td>0.000</td>
</tr>
<tr>
<td>CTA Global</td>
<td>8.071</td>
<td>8.706</td>
<td>0.134</td>
<td>2.887</td>
<td>0.539</td>
<td>0.764</td>
</tr>
<tr>
<td>Distressed Securities</td>
<td>9.973</td>
<td>6.356</td>
<td>-1.675</td>
<td>9.439</td>
<td>333.630</td>
<td>0.000</td>
</tr>
<tr>
<td>Emerging Markets</td>
<td>10.357</td>
<td>13.361</td>
<td>-1.258</td>
<td>8.103</td>
<td>204.960</td>
<td>0.000</td>
</tr>
<tr>
<td>Equity Market Neutral</td>
<td>7.446</td>
<td>3.120</td>
<td>-2.748</td>
<td>20.407</td>
<td>2110.300</td>
<td>0.000</td>
</tr>
<tr>
<td>Event Driven</td>
<td>9.540</td>
<td>6.357</td>
<td>-1.718</td>
<td>9.113</td>
<td>311.480</td>
<td>0.000</td>
</tr>
<tr>
<td>Fixed Income Arbitrage</td>
<td>5.197</td>
<td>4.909</td>
<td>-3.707</td>
<td>22.510</td>
<td>2758.900</td>
<td>0.000</td>
</tr>
<tr>
<td>Global Macro</td>
<td>9.606</td>
<td>5.896</td>
<td>0.815</td>
<td>4.766</td>
<td>36.586</td>
<td>0.000</td>
</tr>
<tr>
<td>Long/Short Equity</td>
<td>9.720</td>
<td>7.681</td>
<td>-0.382</td>
<td>4.247</td>
<td>13.533</td>
<td>0.001</td>
</tr>
<tr>
<td>Merger Arbitrage</td>
<td>8.453</td>
<td>3.869</td>
<td>-1.647</td>
<td>8.793</td>
<td>281.310</td>
<td>0.000</td>
</tr>
<tr>
<td>Relative Value</td>
<td>8.345</td>
<td>4.571</td>
<td>-2.102</td>
<td>12.165</td>
<td>643.860</td>
<td>0.000</td>
</tr>
<tr>
<td>Short Selling</td>
<td>5.109</td>
<td>19.087</td>
<td>0.578</td>
<td>5.249</td>
<td>40.479</td>
<td>0.000</td>
</tr>
<tr>
<td>Funds of Funds</td>
<td>7.338</td>
<td>6.309</td>
<td>-0.459</td>
<td>6.299</td>
<td>74.287</td>
<td>0.000</td>
</tr>
</tbody>
</table>

This table displays the percentage annualised arithmetic mean (first column), the percentage annualised volatility (second column), skewness and kurtosis (in the third and fourth columns, respectively) for hedge fund index returns over the sample period. The last two columns display the Jarque-Bera test and its p-value. We find that most hedge fund strategy returns, with the notable exception of CTA Global, are negatively skewed with a positive excess kurtosis. At the 1% significance level, we reject the null hypothesis that the returns are normally distributed for all strategies except CTA Global. These results confirm that hedge fund returns cannot be assumed to be normally distributed, and consequently that mean-variance analysis is not appropriate for constructing hedge fund portfolios.

3.2 Empirical Protocol

Our objective is to analyse from an out-of-sample perspective the impact of incorporating higher moments (versus simple mean-variance analysis) on portfolio allocation as well as the impact of using improved (versus sample) estimates for coskewness and cokurtosis parameters. To do so, we consider a strategy based on a rolling 24-month calibration period and a three-month holding period. In order to find the optimal weights, we need to estimate \( \mu, M_2, M_3, \) and \( M_4 \) beforehand for each calibration period. As explained in the previous section, we estimate these quantities using two different approaches, namely (i) the sample approach and (ii) the shrinkage approach.

In the sample approach, we estimate all the moments (mean, variance, skewness, and kurtosis) and comoments (covariance, coskewness, and cokurtosis) from the sample estimators defined as follows:

3 - For more information, see www.edhec-risk.com, where monthly data on EDHEC Alternative Indexes can be downloaded.
4 - It is worth stressing that single hedge funds following these strategies might obviously present somewhat different risk-return characteristics.
3. Empirical Analysis

\[ \hat{\mu}_t = E[R_t] = \frac{1}{T} \sum_{t=1}^{T} R_{it} \]

\[ \hat{\sigma}_2^{(i)} = \frac{T}{T-1} \sum_{t=1}^{T} (R_{it} - E[R_i])(R_{jt} - E[R_j]) \]

\[ \hat{s}_{ij} = \frac{T}{T-1} \sum_{t=1}^{T} (R_{it} - E[R_i])(R_{jt} - E[R_j]) \]

\[ \hat{k}_{ijkl} = \frac{T(T-1)(T-2)}{T-3} \sum_{t=1}^{T} (R_{it} - E[R_i])(R_{jt} - E[R_j])(R_{kt} - E[R_k])(R_{lt} - E[R_l]) \]

where \( R_{it} \) denotes the return on hedge fund strategy \( i \) for the month \( t \), \( T \) is the number of observations in the in-sample period, \( \hat{\mu}_i \) is the sample mean return estimator for strategy \( i \), \( \hat{\sigma}_2^{(i)} \) is the sample estimator for the covariance between returns on strategies \( i \) and \( j \), \( \hat{s}_{ij} \) is the sample estimator for the coskewness between returns on strategies \( i \), \( j \), and \( k \), and \( \hat{k}_{ijkl} \) is the sample estimator for the cokurtosis between returns on strategies \( i \), \( j \), \( k \), and \( l \).

In the shrinkage approach, we use the James-Stein shrinkage estimator towards the grand mean for the mean return \( \hat{\mu}_i \) the sample mean return estimator for strategy \( i \), \( \hat{\sigma}_2^{(i)} \) is the sample estimator for the covariance between returns on strategies \( i \) and \( j \), \( \hat{s}_{ij} \) is the sample estimator for the coskewness between returns on strategies \( i \), \( j \), and \( k \), and \( \hat{k}_{ijkl} \) is the sample estimator for the cokurtosis between returns on strategies \( i \), \( j \), \( k \), and \( l \).

In the end, we obtain returns for 126 monthly out-of-sample observations dates for four different optimal portfolios, namely: (a) optimal portfolio based on two moments, using the sample approach for the estimation of the moments and comoments, (b) optimal portfolio based on the four moment approximation of expected utility, using the sample approach for the estimation of the moments and comoments, (c) optimal portfolio based on a four moment approximation of expected utility, using the shrinkage approach for the estimation of the moments and comoments, (d) optimal portfolio based on four-moments and using the shrinkage approach for the estimation of the mean, and (e) optimal portfolio based on four-moments and using the sample approach for the estimation of the mean.
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covariance, coskewness and cokurtosis parameters.

3.3 Empirical Results

To compare the performance of the portfolios built under the aforementioned procedures, we compare certainty equivalents for an investment in the various competing portfolios. More precisely, we follow Ang and Bekaert (2002) and define the economic loss of investing in the portfolio built from sample estimates as the monetary payment the investor requires to be indifferent between sticking to this portfolio and switching to a portfolio based upon improved shrinkage estimates. In other words, this quantity represents the excess amount that must be invested in a portfolio composed using the sample estimates in such a way that the resulting welfare (in terms of expected utility) is identical to the welfare achieved with a portfolio composed using the shrinkage estimates. Accordingly, we solve the following equation for $\bar{W}$:

$$
\frac{1}{T} \sum_{t=1}^{T} e^{-\lambda (1+r_t^{(SCC)})} = \frac{1}{T} \sum_{t=1}^{T} e^{-\lambda W_t(1+r_t^{(S)})}
$$

where the superscript SCC signals the use of a shrinkage towards the constant correlation estimator, superscript S signals the use of the sample estimator and $r_t$ are annualized geometric returns. The annual required payment (monetary utility gain or loss ($MUG/MUL$)) is then given as $MUG = \bar{W} - 1$.

Table 2 reports the annual percentage values for the Monetary Utility Gain/Loss for different levels of risk-aversion $\lambda$.

The following comments are in order. We first consider the numbers in the first column of the first panel labeled "MUG of SCC versus Sample", which represent the $MUG$ obtained from using the shrinkage estimators versus using the sample estimators in a second-order expansion of expected utility, that is, when using mean-variance analysis. These numbers are all positive, suggesting that using shrinkage estimators is an improvement over using sample estimates in mean-variance analysis, which confirms the empirical results in Ledoit and Wolf (2003). Additionally, we find that the $MUG$ increase with the risk-aversion coefficient, which was to be expected since an increase in $\lambda$ indicates a higher weight given to risk parameters, which in turn implies that the impact of any improvement of the risk estimators will then be magnified. We then consider

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>Second-order</th>
<th>Fourth-order</th>
<th>Sample</th>
<th>Shrinkage</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.40</td>
<td>0.40</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td>1.77</td>
<td>1.90</td>
<td>0.10</td>
<td>0.23</td>
</tr>
<tr>
<td>20</td>
<td>8.33</td>
<td>12.98</td>
<td>-2.02</td>
<td>1.82</td>
</tr>
<tr>
<td>30</td>
<td>12.89</td>
<td>13.75</td>
<td>6.86</td>
<td>7.53</td>
</tr>
</tbody>
</table>

The column labeled "second-order" presents the MUG obtained when we use the shrinkage estimators versus the sample estimators in a second-order expansion of expected utility (that is when using mean-variance analysis). The column labeled "fourth-order" presents the MUG obtained when we use the shrinkage estimators versus the sample estimators in a fourth-order expansion of expected utility incorporating skewness and kurtosis. The column labeled "Sample" presents the MUG obtained from using the fourth-order versus second-order expansion of expected utility when using the sample estimators. The column labeled "Shrinkage" presents the MUG obtained from using the fourth-order versus second-order expansion of expected utility when using the shrinkage estimators.
the figures in the second column of the first panel labeled "MUG of SCC versus Sample", which represent the MUG obtained using the shrinkage estimators versus using the sample estimators in a fourth-order expansion of expected utility, that is when moving beyond mean-variance analysis. Again, these numbers are all positive and increasing with $\lambda$, suggesting that using shrinkage estimators is also an improvement over using sample estimates when incorporating higher-order moments in portfolio selection techniques, and that this improvement increases with risk aversion. Moreover, we find that these figures are higher than what is obtained in the context of mean-variance analysis. This result is consistent with similar findings reported by Martellini and Ziemann (2010) for equity portfolio optimization, and suggests that using improved estimators is critical for hedge fund returns where deviations from normality are particularly strong.

The previous analysis has unambiguously shown that an investor who focuses on maximising expected utility of terminal wealth based on a fourth-order approximation of the utility function would benefit from employing improved, as opposed to naïve, estimates for covariance, coskewness, and cokurtosis parameters. On the other hand, a question that remains unanswered by the previous analysis is whether using a fourth-order approximation of the utility function always leads to a better portfolio than to using a second-order approximation. In fact, while it must be the case that using a fourth-order approximation of the utility function generates better results than using a second-order approximation from an ex post perspective (i.e., after removing all uncertainty regarding parameter estimates), it is not necessarily the case that the same holds true ex ante, especially for small sample sizes, owing to the presence of significantly greater estimation risk in the former situation than in the latter situation. To analyse this question, we consider the monetary utility gains (MUG) (or monetary utility losses when negative) implied by moving away from mean-variance analysis based on naïve sample estimates (see results in the first column of the second panel labeled "MUG of fourth-order versus second-order") or shrinkage estimators (see results in the second column of the second panel labeled "MUG of fourth-order versus second-order"). Interestingly, the MUG obtained from using the fourth-order versus second-order expansion of expected utility when using the sample estimators are found to be weaker and occasionally even negative when the risk-aversion parameter is equal to twenty. In fact, it is only when shrinkage estimators are used that portfolio selection with higher-order moments consistently dominates mean-variance analysis from an out-of-sample perspective. Overall our results suggest that improving investor welfare by moving beyond mean-variance analysis, which is justified for hedge fund returns that are strongly non-Gaussian, can be achieved only when robust improved estimators for the coskewness and cokurtosis parameters are used. In the absence of such improved estimators, sticking to mean-variance analysis might prove to be the right option from an ex post perspective because of a lower concern over the robustness in parameter estimates, even though it is suboptimal from an ex ante perspective.
3. Empirical Analysis
4. Conclusion
4. Conclusion

This paper presents an application of the improved estimators for higher-order comoment parameters, recently introduced by Martellini and Ziemann (2010), in the context of hedge fund portfolio optimisation. We find that the use of these enhanced estimates generates significant improvement for investors in hedge funds. We also find that it is only when improved estimators are used that portfolio selection with higher-order moments consistently dominates mean-variance analysis from an out-of-sample perspective. Our results have important potential implications for hedge fund investors and fund of hedge fund managers who routinely use portfolio optimization incorporating higher moments without a formal analysis of the induced increase in parameter uncertainty and related lack of robustness of the results.
5. References
5. References


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About EDHEC-Risk Institute
About EDHEC-Risk Institute

The Choice of Asset Allocation and Risk Management
EDHEC-Risk structures all of its research work around asset allocation and risk management. This issue corresponds to a genuine expectation from the market. On the one hand, the prevailing stock market situation in recent years has shown the limitations of diversification alone as a risk management technique and the usefulness of approaches based on dynamic portfolio allocation. On the other, the appearance of new asset classes (hedge funds, private equity, real assets), with risk profiles that are very different from those of the traditional investment universe, constitutes a new opportunity and challenge for the implementation of allocation in an asset management or asset/liability management context. This strategic choice is applied to all of the centre's research programmes, whether they involve proposing new methods of strategic allocation, which integrate the alternative class; taking extreme risks into account in portfolio construction; studying the usefulness of derivatives in implementing Asset/Liability management approaches; or orienting the concept of dynamic "core-satellite" investment management in the framework of absolute return or target-date funds.

An Applied Research Approach
In an attempt to ensure that the research it carries out is truly applicable, EDHEC has implemented a dual validation system for the work of EDHEC-Risk. All research work must be part of a research programme, the relevance and goals of which have been validated from both an academic and a business viewpoint by EDHEC Risk's advisory board. This board is made up of internationally recognised researchers, the centre's business partners and representatives of major international institutional investors. The management of the research programmes respects a rigorous validation process, which guarantees the scientific quality and the operational usefulness of the programmes.

Six research programmes have been undertaken:
• Asset allocation and alternative diversification
• Style and performance analysis
• Indices and benchmarking
• Operational risks and performance
• Asset allocation and derivative instruments
• ALM and asset management

These programmes receive the support of a large number of financial companies. The results of the research programmes are disseminated through the three EDHEC-Risk locations in London, Nice, and Singapore.

In addition, EDHEC-Risk has developed close partnerships with a small number of sponsors within the framework of research chairs. These research chairs involve a three-year commitment by EDHEC-Risk and the sponsor to research themes on which the parties to the chair have agreed.
About EDHEC-Risk Institute

The following research chairs have been endowed:

- Regulation and Institutional Investment, in partnership with AXA Investment Managers (AXA IM)
- Asset/Liability Management and Institutional Investment Management, in partnership with BNP Paribas Investment Partners
- Risk and Regulation in the European Fund Management Industry, in partnership with CACEIS
- Structured Products and Derivative Instruments, sponsored by the French Banking Federation (FBF)
- Private Asset/Liability Management, in partnership with ORTEC Finance
- Dynamic Allocation Models and New Forms of Target-Date Funds, in partnership with UFG
- Advanced Modelling for Alternative Investments, in partnership with Newedge Prime Brokerage
- Asset/Liability Management Techniques for Sovereign Wealth Fund Management, in partnership with Deutsche Bank
- Core-Satellite and ETF Investment, in partnership with Amundi ETF
- The Case for Inflation-Linked Bonds: Issuers’ and Investors’ Perspectives, in partnership with Rothschild & Cie

Each year, EDHEC-Risk organises a major international conference for institutional investors and investment management professionals with a view to presenting the results of its research: EDHEC-Risk Institutional Days.

EDHEC also provides professionals with access to its website, www.edhec-risk.com, which is entirely devoted to international asset management research. The website, which has more than 35,000 regular visitors, is aimed at professionals who wish to benefit from EDHEC’s analysis and expertise in the area of applied portfolio management research. Its monthly newsletter is distributed to more than 400,000 readers.

| EDHEC-Risk Institute: Key Figures, 2008-2009 |
|-----------------|--------|
| Number of permanent staff | 47 |
| Number of research associates | 17 |
| Number of affiliate professors | 5 |
| Overall budget | €8,700,000 |
| External financing | €5,900,000 |
| Number of conference delegates | 1,950 |
| Number of participants at EDHEC-Risk Executive Education seminars | 371 |

Research for Business

EDHEC-Risk’s activities have also given rise to executive education and research service offshoots.

EDHEC-Risk’s executive education programmes help investment professionals to upgrade their skills with advanced risk and asset management training across traditional and alternative classes.
About EDHEC-Risk Institute

The EDHEC-Risk Institute PhD in Finance
nThe EDHEC-Risk Institute PhD in Finance at EDHEC Business School is designed for professionals who aspire to higher intellectual levels and aim to redefine the investment banking and asset management industries. It is offered in two tracks: a residential track for high-potential graduate students, who hold part-time positions at EDHEC Business School, and an executive track for practitioners who keep their full-time jobs. Drawing its faculty from the world’s best universities and enjoying the support of the research centre with the greatest impact on the European financial industry, the EDHEC-Risk Institute PhD in Finance creates an extraordinary platform for professional development and industry innovation.

The EDHEC-Risk Institute MSc in Risk and Investment Management
nThe EDHEC-Risk Institute Executive MSc in Risk and Investment Management is designed for professionals in the investment management industry who wish to progress, or maintain leadership in their field, and for other finance practitioners who are contemplating lateral moves. It appeals to senior executives, investment and risk managers or advisors, and analysts. This postgraduate programme is designed to be completed in seventeen months of part-time study and is formatted to be compatible with professional schedules.

The programme has two tracks: an executive track for practitioners with significant investment management experience and an apprenticeship track for selected high-potential graduate students who have recently joined the industry. The programme is offered in Asia—from Singapore—and in Europe—from London and Nice.

FTSE EDHEC-Risk Efficient Indices

FTSE Group, the award winning global index provider, and EDHEC-Risk Institute launched the first set of FTSE EDHEC Risk Efficient Indices at the beginning of 2010. Initially offered for the UK, the Eurobloc, the USA, Developed Asia-Pacific, ex. Japan, and Japan, the index series aims to capture equity market returns with an improved risk/reward efficiency compared to cap-weighted indices. The weighting of the portfolio of constituents achieves the highest possible return-to-risk efficiency by maximising the Sharpe ratio (the reward of an investment per unit of risk).

EDHEC-Risk Alternative Indexes

The different hedge fund indexes available on the market are computed from different data, according to diverse fund selection criteria and index construction methods; they unsurprisingly tell very different stories. Challenged by this heterogeneity, investors cannot rely on competing hedge fund indexes to obtain a “true and fair” view of performance and are at a loss when selecting benchmarks. To address this issue, EDHEC-Risk was the first to launch composite hedge fund strategy indexes as early as 2003.

The thirteen EDHEC-Risk Alternative Indexes are published monthly on www.edhec-risk.com and are freely available to managers and investors.
About Newedge
Newedge is a leading force in global multi-asset brokerage with a presence in more than 20 locations across 17 countries and access to more than 85 derivatives and stock exchanges. Supported by two top tier shareholders, Société Générale and Credit Agricole CIB, Newedge is independently run and offers a full range of execution, clearing and prime brokerage services.

Newedge Prime Brokerage, led by Philippe Teilhard de Chardin, has become a leading independent prime broker. It offers global hedge funds a multi-asset class prime brokerage platform covering a variety of assets and instruments across equities, fixed income, foreign exchange and commodities, cash and derivatives.

Newedge Prime Brokerage has received numerous awards including the 2010 Best Prime Broker – Capital Introduction Award from HFMWeek and a top 5 ranking from Global Custodian for European Prime Brokers. In a recent Eurohedge report, Newedge ranked No. 1 in European Global Macro/Fixed Income/CTA mandates and in New Fund Launches.

• **Multi-Asset Prime Brokerage**
Newedge offers clients a single, multi-asset class prime brokerage platform that covers a variety of financial assets including listed products, equities, fixed income, interest rates and foreign exchange. Newedge is committed to helping their hedge fund clients maximize efficiency in trading multiple asset classes, while minimizing operational risks.

• **Capital Introduction**
Newedge’s knowledge and depth of experience allows for introductions to sophisticated investors where a match in criteria is found. Newedge specialises by investor type with dedicated coverage for pension funds, Middle Eastern investors, sophisticated global investors and family offices. Capital introductions are subject to local rules and practices.

• **Financing, Risk Management and Reporting**
Newedge Prime Brokerage offers financing and reporting options that are specifically tailored to client requirements and the different assets and instruments traded. They analyse a customer’s requirements, legal structure and business before providing a personalized risk management solution.

• **Research**
Newedge Prime Brokerage is an innovator in providing investors with benchmarking tools that accurately represent key hedge fund strategy styles. The group also produces an exclusive series of industry-leading research reports and the provision of quantitative statistics on hedge fund return information to investors.

• **UCITS**
For investors looking to allocate capital to liquid and transparent fund structures, Newedge Prime Brokerage is able to offer an open architecture UCITS compliant servicing platform. This allows investors to access best-of-breed hedge funds and CTAs that offer a UCITS compliant investment solution within their product range.

• **Managed Account Services**
Newedge Prime Brokerage provides an independent managed account service enabling investors to access the performance of hedge fund strategies.
About Newedge

through a separately managed account. The service offering allows investors full ownership and control of their assets, and flexible liquidity with daily transparency through customisable reporting.

• Sharia Compliant

For Islamic investors and product organizers such as fund-of-funds, Newedge’s Sharia-compliant servicing platforms provide access to Sharia-compliant alternative investments with a return objective similar to broad equity investments and a volatility objective similar to a broad bond index.
About Newedge
EDHEC-Risk Institute
Publications and Position
EDHEC-Risk Institute Publications (2007-2010)

2010

- Sender, S. EDHEC Survey of the asset and liability management practices of european pension funds (June).
- Amenc, N., F. Goltz, and A. Grigoriu. Risk control through dynamic core-satellite portfolios of ETFs: Applications to absolute return funds and tactical asset allocation (January).
- Goltz, F., and V. Le Sourd. Does finance theory make the case for capitalisation-weighted indexing? (January)

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- Sender, S. The European pension fund industry again beset by deficits (May).
- Le Sourd, V. Hedge fund performance in 2008 (February).
- La gestion indicielle dans l’immobilier et l’indice EDHEC IEIF Immobilier d’Entreprise France (February).
- Real estate indexing and the EDHEC IEIF Commercial Property (France) Index (February).

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• Amenc, N., L. Martellini, and V. Ziemann. Alternative investments for institutional investors: Risk budgeting techniques in asset management and Asset/Liability management (December).
• Goltz, F., and D. Schroeder. Hedge fund reporting survey (November).
• Amenc, N., and D. Schroeder. The pros and cons of passive hedge fund replication (October).
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## Notes

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