Asset Allocation and Asset Pricing in the Face of Systemic Risk: A Literature Overview and Assessment

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Abstract

This paper provides a detailed overview of the current research linking systemic risk, financial crises and contagion effects among assets on the one hand with asset allocation and asset pricing theory on the other hand. Based on the ample literature about definitions, measurement and properties of systemic risk, I derive some elementary ingredients for models of financial contagion and assess the current state of knowledge about asset allocation and asset pricing with explicit focus on systemic risk. The paper closes with a brief outlook on future research possibilities and some recommendations for the further development of capital market models incorporating financial contagion.

Keywords: Asset Pricing, Asset Allocation, Systemic Risk

JEL: G01, G11, G12

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The tradeoff between risk and return has been at the center of financial research for decades. Since the pioneering work of Markowitz (1952) and Sharpe (1964), Lintner (1965) and Mossin (1966), both asset allocation and general equilibrium asset pricing have experienced a rapid development. Generations of researchers as well as practitioners have assessed the qualities of assets, estimated underlying return distributions, computed meaningful performance measures, constructed well-diversified portfolios and deduced fair values of assets.

Inspired by the mean-variance approach, the early literature on the tradeoff between risk and return was mainly based on the consensus that the normal distribution provides a fairly close approximation of the risk inherent in financial assets.\(^1\) Things have changed however, and with the development of more sophisticated capital market models, a different perception of risk has evolved. Nowadays, a large focus is laid on the correct depiction of the tails of a return distribution. The advancements in option pricing theory during the 1990’s using stochastic volatilities (see Heston (1993)) and jumps (see Bakshi, Cao, and Chen (1997) or Bates (2000)) provide representative examples of this altered point of view.

Surprisingly, however, after decades of research, the predominant definition of adverse events still varies among several areas of finance. The asset allocation and asset pricing literature focusses, e.g., on changes in long-run consumption growth (see Bansal and Yaron (2004)) or on large consumption disasters (see Rietz (1988) and Barro (2006, 2009)). On the other hand, regulators, financial intermediaries, central banks and international policymakers have conceived another category of disastrous events: systemic risk.

Loosely speaking, the risk of a collapse of an entire financial system or market is a problem which is as old as the financial sector itself.\(^2\) Tightly linked is the notion of financial contagion which captures the dynamic nature of systemic risk. Adverse events spread out across the economy like a contagious disease, and companies, industries, or countries experience economic shocks without any fault of their own and although they seemed

\(^1\)or, more generally, that the mean and the variance of returns are sufficient key figures of an asset

\(^2\)see, e.g., the large database of international capital flows during the past 200 years presented by Kaminsky (2010)
to be healthy and stable immediately before. The nature of systemic risk and financial crises is catchily summarized in the preface of the comprehensive book on financial crises written by Allen and Gale (2009):

"They are nothing new. They have not been restricted to emerging economies even in recent times. The Scandinavian crises of the early 1990’s are examples of this. Despite having sophisticated economies and institutions, Norway, Sweden and Finland all had severe crises. These were similar in many ways to what happened in the Asian crisis of 1997. Banks collapsed, asset prices plunged, currencies came under attack and their value fell. Output was severely affected."

Since the risk of a financial crisis has always been present and will keep on threatening financial markets in the future, the natural question arises how the existence of systemic risk affects asset allocation and asset pricing results. During the past few years and, of course, enhanced by the subprime crisis starting in 2007, some effort has been made here. The goal of my paper is to provide a detailed overview of the recent literature that links systemic risk, financial crises and contagion effects among assets on the one hand with asset allocation and general equilibrium asset pricing theory on the other hand. Based on the literature about definitions, measurement and properties of systemic risk, I review and assess the existing capital market models which explicitly incorporate financial contagion. The paper closes with a brief outlook on future research possibilities and some recommendations for the further development of models that may quantify the impact of systemic risk on asset allocation and asset pricing.
1 What is systemic risk?

Before one can evaluate the impact of systemic risk on asset allocation and asset pricing, a first – quite natural – question has to be posed: what risk are we actually talking about?

A feasible answer is far from trivial. A promising starting ground for a discussion about financial contagion can be provided by Stijn Claessens, one of the leading researchers in this area of finance who spent several years as an economist at the World Bank and the International Monetary Fund.3

"Contagion refers to the spread of market disturbances – mostly on the downside – from one country to the other, a process observed through comovements in exchange rates, stock prices, sovereign spreads, and capital flows."

Due to his professional background, Claessens confines his definition to contagion in a narrower sense, i.e. contagion among countries, and macroeconomics is indeed the field of research where the term contagion has been coined. The recent subprime crisis (as many financial crises before), however, shows that contagion effects show up in all kinds of financial markets.4

Naturally, the way from an economic definition towards a viable model-theoretic notion of systemic risk seems to be far. The research literature along this way can generally be divided into two strands. The first one addresses the origin of systemic risk and tries to find model-theoretic explanations for the existence of contagion effects in financial markets. The second strand deals with the empirical properties of contagion which can be extracted from financial market data.

Researchers from the model-theoretic first direction typically focus on the market microstructure and the interlinkages among financial institutions and further market participants. Small shocks in one part of the model economy can have the potential to spread

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3 see Claessens, Dornbusch, and Park (2000)

4 More precisely, the term contagion has been coined by macroeconomists whereas the term systemic risk mainly stems from the literature on financial intermediation and regulation. Because of the obvious link between systemic risk, financial crises and financial contagion, all these terms will be used synonymously in the following.
out, e.g. via liquidity and credit problems. Network effects, interdependencies between agents or asymmetric information trigger chain reactions and cause broader financial difficulties in the model economy. A representative example of this literature is the article of Kyle and Xiong (2001) with three types of traders. In their model, traders who generally stabilize the market during normal times may lose money upon a crash and liquidate their positions which, in turn, leads to contagion effects across the whole market. In contrast, Yuan (2005) explains contagion in a macroeconomic rational expectations general equilibrium model with information asymmetry and borrowing constraints.

Since the goal of my paper is to provide an overview of asset pricing and asset allocation research, I will essentially skip this first strand of literature dealing with the correct design of triggers and transmission mechanisms. A very good survey has been written by Allen, Babus, and Carletti (2009). Furthermore, the book of Allen and Gale (2009) stands representatively for the broad work of Franklin Allen, a leading researcher in this area. Last, but not least, a comprehensive summary is again given by Claessens, Dornbusch, and Park (2000):

"The causes of contagion can be divided conceptually into two categories. The first category emphasizes spillovers that result from the normal interdependence among market economies. This interdependence means that shocks, whether of a global or a local nature, can be transmitted across countries because of their real and financial linkages. […] The second category involves a financial crisis that is not linked to observed changes in macroeconomic or other fundamentals but is solely the result of the behavior of investors or other financial agents. Under this definition, contagion arises when a comovement occurs, even when there are no global shocks and interdependence and fundamentals are not factors. […] This type of contagion is often said to be caused by "irrational" phenomena, such as financial panics, herd behavior, loss of confidence, and increased risk aversion."

The large amount of model-theoretic literature providing intrinsic explanations of
systemic risk suggests that contagion is just another empirical fact that needs to be explained by finance researchers. However, this is by no means a generally accepted truth. The second strand of literature on financial contagion deals with the empirical properties of contagion, and many papers in this area indeed address the question whether contagion effects exist at all. Other researchers analyze centuries of financial market data and back out the transmission mechanisms and characteristics of systemic risk from historical financial crises. The insightful anthology of Claessens and Forbes (2001) contains an impressive collection of such empirical investigations. Moreover, it provides a comprehensive overview of the financial crises of the last century and specifies the mechanisms which propagated financial shocks around the globe.

A large part of the empirical literature on the detection of contagion employs normal approximations of return distributions in the spirit of Markowitz (1952) and his successors. Longin and Solnik (2001), among others, analyze the time-varying behavior of international equity market correlations and find that the interdependence increases in bear markets, but not in bull markets. Boyson, Stahel, and Stulz (2010) perform similar econometric studies with special emphasis laid on return contagion across hedge funds. In contrast, Forbes and Rigobon (2002) argue that such tests are biased due to heteroskedastic returns and that correlation coefficients are conditional on market volatility. In their opinion, adjusting for this bias destroys any evidence of contagion effects during the 90’s. On the other hand, these findings are attacked by Corsetti, Pericoli, and Sbracia (2005) who estimate a model with structural breaks in financial transmission mechanisms during crises. Ang and Chen (2002) find that correlations are much greater for downside moves, especially for extreme downside moves, than for upside moves. To summarize, the empirical evidence at least indicates that volatilities and correlations change in times of financial distress. Nevertheless, the following fundamental critique by Karolyi (2003) seems appropriate:

"One of the limitations of the existing literature on contagion has been its focus on correlations of asset prices. Few of the concerns expressed about contagion seem to be based on measures of association. Rather, the concerns
seem to be founded on some presumption that there is something different about extremely bad events that might lead to irrational outcomes, excess volatility and panic that grip investors. The problem with correlations is that they give equal consideration to small and large price changes which precludes an evaluation of the special impact of large changes.”

Consequently, Bae, Karolyi, and Stulz (2003) put a strong focus on tail risk and analyze the coincidence of extreme return shocks across countries within regions and across regions. Similarly, Pukthuanthong and Roll (2010) discuss various jump measures and investigate how jumps in equity indexes all around the world are correlated. Last, but not least, Markwat, Kole, and van Dijk (2009) concentrate on the notion of domino effects. They find evidence that global crashes do not occur abruptly, but are preceded by local and regional crashes, and they show that global crashes are, to a certain extent, predictable.

A thoughtful reflection of the econometric arguments which is still valid today is given by Claessens and Forbes (2004). And although the econometric method to study systemic risk is discussable, Allen and Gale (2009), Claessens and Forbes (2004) and Pericoli and Sbracia (2003), among others, extract some general characteristics of contagion from the intense debate sketched above. In the following, I will exploit these characteristics to assess the current literature linking systemic risk with asset pricing and asset allocation.

First of all, contagion can (but need not) be triggered by an initial shock. This may be a reduction in global economic growth, a change in commodity prices, or a change in interest rates or currency values. Any of these shocks can lead to increased comovement in capital flows and asset prices.

Secondly, there can be a fundamental transmission mechanism such that other institutions in the system are affected in the aftermath of the initial shock. Examples are trade linkages or financial linkages among economies. Shocks may, however, also be transmitted without any obvious linkages, e.g. due to investor irrationality. Investor behavior may be individually rational ex-ante, but can lead to excessive comovement in market prices, in
the sense that market prices are not explained by real fundamentals.

Thirdly, whether or not propagated through market linkages, systemic risk goes along with a change in the (perceived) risk-return tradeoff in the economy. This implies that contagion has a time dimension, the risk of adverse events in financial markets is increased for a certain time period. Stated differently in terms of the asset allocation and asset pricing literature, systemic risk influences the investment opportunity set.

Fourthly, contagion does not only manifest in the degree of risk in the economy, but may also be accompanied by a large amount of uncertainty. Market participants do not only fear subsequent losses in some asset prices, but there is also a large dispersion about the magnitude of a current crisis. Investors may, e.g., have different beliefs about future growth rates in the economy so that very pessimistic traders seem to behave irrationally from the point of view of rather optimistic traders.

Finally, to a large extent, systemic risk refers to the tails of return distributions. As the econometric literature shows, it is quite hard to perform powerful unbiased econometric tests for structural breaks in market correlations. On the other hand, defining contagion via the coincidence of extreme events in financial markets seems to provide ample empirical evidence of systemic risk.

2 How to transfer the empirical properties into a tractable economic model

Having established the basic properties of systemic risk, the next step is to construct a capital market model which is suitable to assess the impact of contagion on asset allocation and asset pricing.

First of all, contagion is, to a large extent, driven by extreme shocks in the return of certain assets. The notion of shocks is well established in the asset allocation and asset pricing literature. Starting from the path-breaking articles of Merton (1969, 1971) who solves the asset allocation problem in a diffusion framework, many researchers have tried to incorporate jump-diffusion processes. Perhaps the most prominent approach is

In the asset pricing literature, jump risk has been introduced by Rietz (1988) and Barro (2006, 2009). These authors solve the equity premium puzzle by adding large, but infrequent consumption disasters to the commonly applied Lucas tree model. Wachter (2010) and Gabaix (2010) explain further asset pricing puzzles by adding recursive utility functions and time-varying disaster intensities. The results of disaster models are, however, recently questioned by Chen, Joslin, and Tran (2010) in an economy with heterogenous agents and risk sharing. Liu, Pan, and Wang (2005) model imperfect information about jump risk and ambiguity aversion and analyze empirically observed option-implied volatility smiles. In another general equilibrium option pricing framework, Bates (2008) adjusts the utility function of investors by a specific crash aversion. Empirically, Backus, Chernov, and Martin (2011) back out the risk-neutral distribution of disasters from observed equity option prices. Bollerslev and Todorov (2011) analyze equity and variance risk premia and find both large and time-varying compensations for fears of disasters.

A further characteristic of contagion is its time dimension. Systemic risk is often accompanied by a change in economic fundamentals. In terms of the asset allocation and asset pricing research, contagion has the potential to change the investment opportunity set. Some asset allocation researchers try to capture time-varying economic fundamentals in so-called regime switching models in which the state space is finite and often driven by an underlying Markov chain. Ang and Bekaert (2002) model regime switching correlations and volatilities, Honda (2003) studies unobservable and regime switching mean returns, and Guidolin and Timmermann (2007, 2008) combine these approaches and construct several bull and bear regimes. Guidolin and Hyde (2010) compare the resulting portfolio policies with simpler VAR strategies and find that explicitly accounting for regime switches significantly increases the expected utility. Last, but not least, Tu (2010) connects regime switching, model uncertainty and parameter uncertainty in an asset allocation framework. Regime switching models are also very common in asset pricing. Recent examples with
jump risk are the articles by Elliott, Miao, and Yu (2008) and Buraschi, Porchia, and Trojani (2010b) in which the intensity of rare disasters follows a Markov chain.

Besides finite state Markov chains, the introduction of state variables with continuous state space has become very popular in recent years. Examples in the asset allocation literature are the papers of Kim and Omberg (1996) with stochastic risk premia as well as Liu, Longstaff, and Pan (2003), Liu and Pan (2003) and Branger, Schlag, and Schneider (2008) with stochastic volatility in a complete and incomplete market. Buraschi, Porchia, and Trojani (2010a) solve a diffusive model with two risky assets whose correlation is stochastic following a so-called Wishart process. A leading role in asset pricing is played by Bansal and Yaron (2004) and their numerous successors who can explain, e.g., the equity premium puzzle via long-run consumption risk and stochastic volatility of consumption growth in a recursive utility framework. In similar models, Wachter (2010) and Gabaix (2010) assume a stochastic disaster intensity in order to solve the excess volatility puzzle and further puzzles in the bond market. Drechsler and Yaron (2011) can reproduce several empirical features of variance risk premia in the framework of Bansal and Yaron (2004).

Another property of systemic risk, namely the uncertainty about current fundamentals driving the economy, is at the heart of an ongoing debate in the asset allocation and asset pricing literature and a bunch of seminal papers has been published here. The mathematical basis is provided by Dothan and Feldman (1986), Detemple (1986) and Hamilton (1989). Recent advances in asset allocation have been made by Brennan (1998) who investigates uncertainty about mean returns, and Xia (2002) who analyzes uncertainty about the predictability of stock returns. Veronesi (2000) clarifies the relationship between uncertainty about economic growth and general equilibrium stock returns. Epstein and Schneider (2008) generalize these results by assuming an ambiguity-averse investor who explicitly dislikes uncertainty. A comprehensive overview on learning in financial markets is given by Pastor and Veronesi (2009).

To summarize, all the aforementioned articles mark important steps in the asset allocation and asset pricing research and their impact on the current level of knowledge in these fields has been enormous. Nevertheless, they do not really incorporate systemic
risk. In a sense, they provide a partial framework for contagion picking one or two of its basic characteristics, but do not focus on the problem as a whole. In order to provide a comprehensive model of contagion, one has to dig a bit deeper. And indeed, ideas how to combine some of the models presented above can be found in the mysterious abyss of mathematical finance. Reconsidering the properties of contagion established in the previous section, systemic risk in financial markets has many similarities to and may be seen as a special case of a much broader class of risk: portfolio credit risk.

The analysis of portfolio credit risk has experienced an enormous growth during the last two decades which is due to – and certainly also enhanced – the evolution of structured credit derivatives like collateralized debt obligations, asset backed securities, and multiname credit default swaps. The development of portfolio credit risk models, e.g. via copula functions, has become a major playing field of applied mathematicians all over the world. David Lando, one of the leading researchers in this field, describes the basic challenge as follows: 6

"Modeling dependence between default events and between credit quality changes is, in practice, one of the biggest challenges of credit risk models. […] Finally, dependence modeling is necessary in trying to understand the risk of simultaneous defaults by, for example, financial institutions. Such simultaneous defaults could affect the stability of the financial system with profound effects on the entire economy."

The research on credit derivatives faces problems quite similar to the ones presented in the previous section. Defaults in a credit portfolio can (but need not) be initialized by defaults in single assets that may trigger contagion in the aftermath. In many cases, there are linkages between credits in a portfolio. A prominent example is the recent subprime crises with the breakdown of numerous synthetic CDOs which were constructed out of other CDOs which again were constructed out of CDOs and so forth. But even

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5 Of course, no one of the authors indeed claims to do so.
6 See Lando (2004)
a single bank can show up some fundamental linkages among its creditors, e.g. due to regional or sectoral specialization. Furthermore, the time dimension of portfolio credit risk is a striking fact of which, again, the recent subprime crisis provides rich examples. As Duan (2010) points out using an enormous database of more than 1.3 million firm-month observations, defaults seem to cluster, i.e. they tend to occur in waves. In particular, copula models can hardly account for this time dimension. Moreover, there can be a large degree of uncertainty about the true amount of risk in a specific credit portfolio. During the subprime crisis, a lack of confidence in banks’ risk management tools, i.e. in their ability to reduce this uncertainty, led to enormous liquidity problems because banks could no longer get funding at the money market. Last, but not least, default risk is by far the most prominent example of tail risk which has been identified as a main characteristic of systemic risk in the previous section as well. To summarize, there are many good reasons to borrow some ideas for the modeling of systemic risk and contagion from the mathematical finance community working on portfolio credit risk.

Similarly to the credit risk literature in general, the existing portfolio credit risk models can be divided into two major categories: structural models and intensity models. Structural credit portfolio models are related to the literature on the evolution of contagion described in the previous section. Authors in this area assume stochastic processes for the evolution of debt and equity of a firm and determine the exact time of a default endogenously. Modifying the stochastic dynamics can then generate a significant amount of default contagion. Giesecke (2004, 2006) and Giesecke and Goldberg (2004) add the important feature of incomplete information and uncertainty about fundamentals to structural credit risk models. Giesecke and Weber (2004) quantify the relation between the variability of global economic fundamentals, strength of local firm interaction, and the fluctuation of losses. Jorion and Zhang (2009) model credit contagion via direct counterparty effects among financial institutions which are engaged in the CDS market. Battiston, Gatti, Gallegati, Greenwald, and Stiglitz (2009) build a network of liabilities among agents as a system of coupled stochastic processes and analyze the tradeoff between risk diversification and the creation of systemic risk. A similar network model is
tested empirically by Cont, Moussa, and Santos (2010) in order to address regulatory is-

sues of the banking system. In further empirical tests of structural models, Duffie, Eckner,
Horel, and Saita (2009) and Duffie, Saita, and Wang (2007) find evidence for the presence
of common latent factors driving default correlation, even when controlling for observable
structural factors which are known to explain default probabilities. The applicability of
structural models in theoretical asset allocation and asset pricing is, however, question-
able because of their great complexity and because the main focus of these models lies on
the explanation of defaults out of fundamental firm data.

The second major category of credit risk literature comprises the so-called intensity
models. The workhorses of these models are counting processes. Usually, default histories
are modeled as realizations of an exogenous Poisson process. In order to introduce time
variation in default risk, researchers employ so-called doubly stochastic Poisson processes,
i.e. they let the intensity of the Poisson process itself follow another stochastic process.
In order to keep the numerical analysis tractable, the intensity of the Poisson processes
is typically governed by a discrete Markov chain and switches between a finite number
of values. One can, e.g., think of quite general covariates like business cycle variables or
crisis indicators driving the default intensities. Additionally, the Markov chain might be
non-observable. An investor would then have to estimate the current state of the Markov
chain out of observations of asset prices. A nice example of such a filter is given in Frey
and Runggaldier (2010). Bielecki, Crépey, and Herbertsson (2011) give a comprehensive
overview of models in which default intensities are controlled by an exogenous or endoge-
nous Markov chain.

Generally, doubly stochastic Poisson processes can reproduce many of the empirical
features of systemic risk since they establish a time dimension of contagion. Davis and
Lo (2001), Focardi and Fabozzi (2005) and Azizpour and Giesecke (2008), among others,
further extend these intensity models towards the notion of contagion explained in the
previous section. In their studies, a default in one asset increases the intensity of the Pois-
son processes of the surviving assets in the credit portfolio. Related to these approaches
are the so-called Hawkes processes propagated by Ait-Sahalia, Cacho-Diaz, and Laeven
(2010). These can be seen as Poisson processes whose intensity is a continuous function of the past realizations of the Poisson process itself. Typically, the intensity jumps up if the Poisson process jumps and decays gradually afterwards. Although path-dependent in general, the intensity can sometimes be reformulated as a Markov process, e.g. if the decaying after a jump is modeled exponentially. Ait-Sahalia, Cacho-Diaz, and Laeven (2010) focus on the empirical estimation of such processes and leave much space for future research in asset pricing and asset allocation. In the paper of Giesecke, Spiliopoulos, and Sowers (2011), the stochastic intensity of each single credit is influenced by an idiosyncratic risk process, a systematic risk process common to all credits and past defaults. In order to meet empirical data on default clustering, Duan (2010) proposes a hierarchical intensity model with common, group-specific and individual shocks.

Regarding the characteristics of systemic risk established in the previous section, the doubly stochastic intensity models seem to provide a promising direction of research. They deal with tail risk – namely default risk – and they can reconstruct the time dimension of contagion since they incorporate state variables which manifest in stochastic jump intensities. Moreover, they can provide a direct trigger for contagion. In the paper of Davis and Lo (2001), a default in one credit can, but need not increase the default intensity of all remaining credits in the portfolio. Last, but not least, intensity models can be extended to incorporate a significant extent of uncertainty. E.g., Davis and Lo (2001) assume that the current default intensities in the portfolio are unobservable and need to be filtered from the history of observed credit events. In the following, I will therefore present the currently available literature on asset allocation and asset pricing with special focus on contagion effects and assess how these studies can be intensified by combining well-known asset allocation and asset pricing techniques presented above with some ideas from portfolio credit risk modeling.
3 Asset allocation: What is the impact of contagion on an investor’s portfolio choice?

The first prominent paper tackling this question is the one by Das and Uppal (2004). The authors assume jump-diffusion processes for all asset prices and add the simple restriction that a jump in the economy must affect all assets jointly at one point in time, i.e. there are no idiosyncratic, but only systemic jumps. They calibrate their model to equity index data from six countries and find that systemic risk reduces the gains from diversification slightly and penalizes highly levered investors sharply. In an extension, Prokopczuk (2011) compares the gains from crisis-conscious and crisis-ignorant investment strategies and finds that the crisis-ignorant strategy performs significantly worse. With constant relative risk aversion utility, the framework is computationally very tractable. However, the model lacks some of the main properties of contagion. The time dimension of contagion is completely neglected and the investment opportunity set is constant. Moreover, there is no uncertainty about the jump distribution or about growth rates. The optimal portfolios obtained from empirical data on index returns are therefore questionable. They do not regard that the investment opportunity set can be stochastic and, thus, do not contain a hedging demand against the risk of changing economic fundamentals affecting the distress probabilities of some assets.

A more promising approach is presented by Kole, Koedijk, and Verbeek (2006). These authors focus on the estimation of regime switching models. In a first step, they extract several regimes from equity index data of seven countries using a diffusive model for the index returns. Identifying one of these regimes – namely the one with the lowest expected returns and highest volatilities of return – as a crash regime, the authors then compare the performance of an investor taking systemic risk into account with the performance of an investor neglecting the crash regime and find that the loss from ignoring the crisis regime can be quite substantial. To summarize, the paper captures the time dimension of contagion and gives further empirical evidence for this property. However, the correct notion of tail risk seems questionable: systemic risk is, once again, defined as an increase
in volatilities and correlations.

In a purely theoretical paper, Ait-Sahalia, Cacho-Diaz, and Hurd (2009) analyze a general model with Lévy processes and several utility specifications. Using an orthogonal decomposition of the n-dimensional space of possible stock returns, they prove a separation theorem. The optimal portfolio strategy is to set the overall exposure to jump risk to the desired level first and, in a second step, to exploit any differences in expected returns and Brownian variances and covariances using the orthogonal decomposition of $\mathbb{R}^n$. The Lévy process assumption allows for a very rich modeling of the tail risk of the underlying assets, e.g. by including jump processes with infinite activity instead of finite activity Poisson processes only, or by including simultaneous jumps in some or even all available assets. However, this paper again lacks some important properties of contagion, namely the time dimension, i.e. the time-varying investment opportunity set, and the possibility of uncertainty about the distribution of returns or about the model parameters.

Another approach to unfold the link between contagion and asset allocation is made by Branger, Kraft, and Meinerding (2009). Extending ideas of Kraft and Steffensen (2009) and Diesinger, Kraft, and Seifried (2010), the authors model a CRRA investor and an economy with two regimes, a calm and a contagion state. The assets follow jump-diffusion processes and the jump intensity is stochastic depending on the current market regime. The regimes are driven by a Markov chain which is itself driven by jumps in all stocks in the economy, i.e. a jump in one of the stocks can, but need not trigger a regime switch and increase all jump intensities in the economy. In a sense, this model is related to the models of portfolio credit risk described in the previous section in various ways.

Economically, the focus of Branger, Kraft, and Meinerding (2009) lies on model risk. The authors show that ignoring contagion or modeling contagion incorrectly, e.g. by using joint jumps similar to Das and Uppal (2004), can reduce the expected utility of an investor substantially. Similarly to Kole, Koedijk, and Verbeek (2006), the authors find that the time dimension of contagion is the most important device in terms of expected utility. Surprisingly, however, the average duration of the different regimes has a minor impact. The mere fact that contagion affects the investment opportunity set, drives the main
results. Moreover, the paper explicitly takes into account that there is a concrete trigger for contagion, in this case a jump in one of the assets. Nevertheless, some aspects of systemic risk are still missing. First of all, the uncertainty coming along with systemic risk is neglected. Secondly, the notion of tail risk is very simple in this paper: the authors assume a constant jump size in order to allow for market completeness and closed-form solutions, and they restrict to two economic regimes only.

In an extension of Branger, Kraft, and Meinerding (2009), Branger, Kraft, and Meinerding (2011a) introduce the aspect of uncertainty into their model. The investor does no longer know the current state of the economy exactly, but has to estimate this state from observed stock prices. A similar asset allocation problem is also solved by Bäuerle and Rieder (2007), however without explicit focus on contagion effects and, therefore, with one risky asset only. Interestingly, the numerical results of Branger, Kraft, and Meinerding (2011a) show that it is more or less sufficient to use the information from observed jumps for the estimation only, i.e. additionally incorporating information from diffusion processes does not add much value. This finding is, of course, only valid if diffusion and jumps can be disentangled with sufficient precision.\footnote{A fact which is, e.g., approved by Ait-Sahalia (2004)} Furthermore, the correct depiction of tail risk is still questionable due to the assumption of constant jump sizes and two market regimes only.

Another attempt to link systemic risk and asset allocation has been made by Ait-Sahalia, Cacho-Diaz, and Laeven (2010). These authors employ a jump-diffusion model with Hawkes processes which are inspired by the existing literature on portfolio credit risk as explained in Section 2. The jump intensities of each asset jump up by a certain amount if one of the assets jumps and mean-revert exponentially afterwards so that the intensity still follows a Markov process. From an economic point of view, the resulting exogenous intensity process is similar to the endogenously generated perceived jump intensity of Branger, Kraft, and Meinerding (2011a). Ait-Sahalia, Cacho-Diaz, and Laeven (2010) show that this specification can generate the empirically observed default clustering. Furthermore, it recaptures the time dimension of systemic risk quite well. Moreover,
in this model, there is always a trigger for a crisis, namely a jump in one of the stock prices which increases the jump intensities of some or all other assets in the economy. On the other hand, the model lacks a proper depiction of uncertainty. And last, but not least, a thorough numerical analysis of the portfolio impact of systemic risk is still missing in this paper. Ait-Sahalia, Cacho-Diaz, and Laeven (2010) indicate how to solve the portfolio problem, but confine themselves to the econometric problem of calibrating the Hawkes processes to current stock market data.

To summarize, some effort has been made to shed light on the link between systemic risk and asset allocation. The economic results are, however, still very scarce. First of all, the time dimension seems to be the most important model feature. An investor ignoring the possibility of stochastic investment opportunities incurs a substantial utility loss as Branger, Kraft, and Meinerding (2009) and Kole, Koedijk, and Verbeek (2006) show. This loss can even exceed the gain from trading in additional instruments like derivatives, i.e. from market completeness. Secondly, an investor with restricted information who has to estimate certain key figures like the current value of the jump intensities incurs a utility loss as well. As Branger, Kraft, and Meinerding (2011a) show, this loss can be as large as the loss from ignoring the time-varying investment opportunities. Further results show that contagion effects have a crucial impact on an investor’s security demands and reduce the gain from diversification.

From a modeling point of view, the most interesting starting ground for future research is provided by Ait-Sahalia, Cacho-Diaz, and Laeven (2010). The rich dynamics which can be generated by Hawkes processes seem to be appropriate to generate the empirically observed default clustering which has been depicted by Duan (2010). However, much work remains to be done.
## 4 General equilibrium asset pricing: What is the market price of systemic risk?

The literature on general equilibrium asset pricing models with special focus on systemic risk can generally be divided into two strands. The first one deals with equilibrium models that endogenously create systemic risk via network effects, asymmetric information or portfolio constraints as pointed out in Section 1. Prominent examples are the papers of Yuan (2005) and Kyle and Xiong (2001) which have already been presented. Another model is set up by Ribeiro and Veronesi (2002) who assume an exchange economy with uncertainty about the drift rates of all dividend processes. The representative investor has constant absolute risk aversion utility and has to estimate the current value of the drift rates which are driven by one common Markov chain. The uncertainty of the investor is time-varying, thereby creating excess comovement during bad times.

The second strand of literature is at the center of this survey and tries to go one step further. The question is no longer how systemic risk can arise in a general equilibrium. Instead, the models assume that systemic risk is present in the economy and try to assess how the possibility of a systemic event influences asset prices, expected returns, volatilities of returns and other key asset pricing figures. This area of financial research is rather new and the major effort has been made here during the past five years.

An asset pricing study about contagion risk on corporate bond markets is performed by Collin-Dufresne, Goldstein, and Helwege (2010). These authors assume an investor with constant relative risk aversion utility. The outputs of each firm are subject to idiosyncratic diffusion risk, systematic diffusion risk, idiosyncratic extreme shocks as well as market-wide production shocks. However, the authors do not compute the pricing kernel explicitly, but instead apply a prespecified discount factor which they deduce from previous asset pricing literature. Their focus lies on the empirical estimation of a contagion risk premium. Calibrating their model to historical data, the authors find that by far the largest part (around 90%) of the default risk premia on corporate bonds is attributable to contagion risk, i.e. to the risk of a sudden large market-wide production shock. The authors also
extend their model by uncertainty about the current default intensities and filtering. Their
results show that surprising credit events of large firms (i.e. when the estimated default
intensity is low) can generate a market-wide increase in credit spreads and a flight-to-
quality response, i.e. a strong demand for relatively safe assets.

As I have pointed out previously, an important feature of contagion and systemic risk
is its time dimension. An obvious idea to bring contagion effects into an equilibrium model
is therefore the introduction of stochastic state variables like the stochastic jump intensity
in the paper of Collin-Dufresne, Goldstein, and Helwege (2010). Unfortunately, in a simple
Lucas (1978) exchange economy with one representative investor and constant relative risk
aversion utility, state variable risk is not priced since the investor has no preference for the
timing of the resolution of uncertainty. A necessary ingredient to introduce state variable
risk into the pricing kernel and create something like a contagion risk premium would
therefore be the use of more sophisticated utility functions, e.g. recursive utility.

A paper which goes in this direction, is the one by Benzoni, Collin-Dufresne, and
Goldstein (2010). These authors assume a Bansal and Yaron (2004) economy with recur-
sive preferences and with additional jumps in the long-run risk and volatility processes.
The jump intensity is driven by a hidden Markov chain with two states so that the in-
vestor has to filter the current jump intensity from the observation of jumps. The authors
assume that the current values of the state variables are observable and that the investor
can perfectly disentangle jumps and diffusion in these state variables. Systemic risk is
induced by a jump in the underlying state variables which explicitly regards the time
dimension of systemic risk. Economically, the authors put a special focus on contagion
effects in option markets. Calibrating their model to historical data, they argue that the
1987 crash was indeed a systemic event: there was a minimal impact on observable eco-
nomic variables like consumption, but the slope of the implied volatility curve changed
dramatically after the crash which is, in their opinion, a result of dynamic learning about
the probability of a systemic event.

In another recursive utility Lucas tree model, Nowotny (2010) employs Hawkes pro-
cesses and self-exciting jumps which have already been explained in the previous section.
Consumption follows a jump-diffusion process. Additionally, the jump intensity jumps up in case of a jump and reverts to a long-run mean afterwards, a setup similar to the one in Ait-Sahalia, Cacho-Diaz, and Laeven (2010). Since the model is still affine, the pricing kernel can be obtained in closed form. Economically, the paper can explain a declining risk-free rate and a declining price-dividend ratio of the risky asset during times of crisis. Furthermore, the equity premium and the model-generated VIX shoot up in response to a disaster. The paper can also account for a significant amount of return predictability and match implied volatility smiles for index options. Nevertheless, the model still lacks an important feature of systemic risk, namely the uncertainty about underlying fundamentals like the jump intensity.

The papers of Benzoni, Collin-Dufresne, and Goldstein (2010) and Nowotny (2010) already seem to quantify the link between asset pricing and contagion quite well. However, one very obvious point is still missing in their analysis. The first natural assumption when discussing a phenomenon like financial contagion should be the one that there is more than one risky asset in the market. Intuitively, one risky asset should be the trigger of a systemic event in the market, and another risky asset – usually different from the first one – should be the one which is contaminated through contagion.

Although self-evident, this simple assumption poses a hard challenge to general equilibrium asset pricing models. Loosely speaking, the reason is that the sum of two lognormally distributed random variables is not lognormally distributed again. In other words, summing up two risky dividend streams following a geometric Brownian motion in a Lucas (1978) exchange economy, the resulting consumption process will not follow a geometric Brownian motion again which makes the computation of the pricing kernel and all other asset pricing figures somewhat burdensome.

For this reason, the literature on Lucas (1978) exchange economies with more than one output tree is relatively scarce, but it has seen some growth in the past few years. The seminal paper in this field is certainly the one of Cochrane, Longstaff, and Santa-Clara (2008). The authors are the first to solve a model with logarithmic utility and two Lucas trees following simple geometric Brownian motions. They show that a lot of
effects simply arise from the market clearing mechanism with two trees instead of one. There are interactions among the prices of the trees despite the fact that the underlying output processes are mutually independent. Expected returns, equity premia and return volatilities are time-varying, returns display serial correlation and are predictable from price-dividend ratios, and the model generates excess volatility of prices and returns.

Martin (2009) extends the framework of Cochrane, Longstaff, and Santa-Clara (2008) to constant relative risk aversion utility, general affine dynamics for the dividend processes and an arbitrary number of trees (which, in his terminology, constitute a so-called Lucas orchard). The main advancement in the paper of Martin (2009) is the semiclosed-form solution for the pricing kernel which makes the model analytically tractable. Branger, Schlag, and Wu (2011) establish a slight extension of Martin (2009) and introduce affine state variables besides the affine dividend processes. An extension to recursive utility functions is conceptually developed by Branger, Dumitrescu, Ivanova, and Schlag (2010).

Since the computational burdens can still be quite cumbersome, besides the Lucas orchard research initiated by Cochrane, Longstaff, and Santa-Clara (2008), there exists a second, concurring strand of literature which builds on the work of Pavlova and Rigobon (2007). Basically, Pavlova and Rigobon (2007) propose a model with two Lucas trees as well. However, they assume that each tree represents the output of one country. Each country has its own representative investor consuming only goods from his home country, the pricing kernel for each country is basically identical to the one tree pricing kernel. In order to construct a general equilibrium, the authors introduce a stochastic exchange rate and deduce several testable hypotheses for international financial markets afterwards. Economically, this paper belongs to the strand of literature which tries to explain contagion effects endogenously rather than scrutinizing its implication for asset prices in general. However, the framework has inspired some researchers working on Lucas orchard exchange economies.

Conceptually, the main difference between these two strands is given by the aggregation of the outputs of the trees. Cochrane, Longstaff, and Santa-Clara (2008) assume that consumption is just the sum of all dividends and the utility of the investor depends
on this aggregate consumption. On the other hand, Pavlova and Rigobon (2007) assume
that the investor has a separate utility function for the output of each tree and aggregates
the utilities rather than the consumption. Unfortunately, a thorough discussion about the
economic implications of the aggregation method has not been published by now. Some
ideas might be taken from a paper by Martin (2010) which tries to draw a conceptual link
between the two approaches.

Since aggregating utilities is much more tractable than aggregating dividends, some
extensions of the work of Pavlova and Rigobon (2007) have been propagated in the past
few years. Buraschi, Trojani, and Vedolin (2010) suggest a model with two trees with
unobservable expected dividend growth rates and two agents with differences in beliefs.
Economically, they address some empirical features of correlation and volatility risk premia.
Vedolin (2009) adds an additional channel of uncertainty, namely uncertainty about
the parameters governing the unobservable stochastic evolution of the drift rates. Econom-
ically, she focusses on the cross-section of volatility risk premia. Berndt (2011) provides
a closed-form solution for an extended model where the agents additionally show up heter-
egenous risk aversion. Pavlova and Rigobon (2008) extend the model of Pavlova and
Rigobon (2007) to three countries and three goods with special emphasis on portfolio con-
straints of investors. Coeurdacier and Guibaud (2009) embed financial frictions like taxes
or imperfectly integrated international stock markets. Last, but not least, Colacito and
Croce (2011) and Colacito and Croce (2010) show how to incorporate recursive utility.
Economically, they focus on international financial markets and exchange rate puzzles.

However, only few Lucas orchard papers explicitly address the aspect of systemic risk.
Among these, perhaps the most promising one is from Buraschi, Porchia, and Trojani
(2010b). They solve the aggregation problem from above neatly by assuming only one
diffusion process for all dividends. Each dividend is defined as the product of this diffusion
with a firm-specific multiple. Each multiple follows a two-state Markov chain where the
low value indicates that a firm is in distress while the high value can be seen as a recovery
state. The transition probabilities for all Markov chains depend on an unobservable latent
factor which, again, follows a two-state Markov chain with known transition probabilities.
Consumption is defined as the sum over all dividends and the investor has constant relative risk aversion preferences so that a closed-form solution for the pricing kernel is possible. In a further extension of the model, the transition probabilities of the latent factor can depend on the multiple of some dividends which provides a rich package of network and feedback effects among the firms. Economically, the authors show that learning helps to generate more realistic dispersion of cross-sectional expected returns. As a result of their asymmetric propagation patterns for contagion, they can model the cross-sectional variability of expected returns in response to dividend shocks and elicit risk premia related to the different degree of distress risk among assets.

Conceptually, the authors capture many salient features of systemic risk and the model is tightly linked to the portfolio credit models presented in Section 2. First of all, the authors explicitly regard the time dimension of contagion. The latent factor driving the Markov chain transition probabilities can produce high default probabilities and low recovery probabilities indicating a crisis regime and converse probabilities indicating normal market times. Moreover, since the factor is unobservable and the investor has to estimate the current state of the economy, there is a significant amount of uncertainty in the model. Whenever a firm is in distress, the fear that this distress could have been caused by bad macroeconomic conditions increases. Furthermore, since the transition probabilities of the latent factor can depend on the dividends of some assets, there is also some kind of trigger for contagion. Assume, e.g., that one of the firms is a large bank. A distress of this bank can increase the overall probability that the economy slides into a crisis. Once the latent factor has switched to the crisis regime, all other default intensities in the economy increase.

On the other hand, the notion of tail risk in this paper is highly questionable. Whenever a firm is in distress, the investor knows for sure that it will recover sooner or later since the Markov chain of each firm has only two possible states. If there was no uncertainty about the latent state of the economy, this would imply that the expected consumption growth automatically jumps up in times of distress. Adding uncertainty to the model reduces this effect since the estimated possibility of further defaults increases in response
Another Lucas orchard paper which explicitly resolves this last concern is the one from Branger, Kraft, and Meinerding (2011b) which is itself an extension of the Martin (2009) framework. The authors of this paper assume jump-diﬀusion dynamics for two Lucas trees and let the jump intensities follow a two-state Markov chain as in Branger, Kraft, and Meinerding (2009). As a special feature, the Markov chain is directly governed by jumps in both dividends. Jumps in the dividends can, but need not increase the jump intensities of all dividends in the economy, i.e. there is a model-endogenous trigger for systemic events. The authors can solve for the pricing kernel in semiclosed form in the spirit of Martin (2009) so that the model is kept analytically tractable.

Economically, Branger, Kraft, and Meinerding (2011b) find signiﬁcantly positive correlations between assets (even if dividends are uncorrelated) and large positive betas for small assets. Furthermore, the authors also ﬁnd effects like ﬂight to quality, i.e. the demand for small contagion-robust assets increases during crises whereas the demand for small contagion-sensitive assets decreases relative to the overall demand for risky assets. Conceptually, the paper of Branger, Kraft, and Meinerding (2011b) proposes a more realistic notion of default risk. Nevertheless, some aspects are still missing, mainly the uncertainty about the true nature of a crisis as in Buraschi, Porchia, and Trojani (2010b).

To summarize, some attempts have already been made to analyze the link between systemic risk and asset pricing ﬁgures. However, a major shortcoming of all the Lucas orchard models above is still the correct notion of default risk. A closer look at the ideas from portfolio credit risk models presented in Section 2 could help get a better idea of modeling default clustering, e.g. by mutually exciting Hawkes processes with uncertainty about the current value of the jump intensities.

Besides this, the results presented above suffer from the assumption of constant relative risk aversion utility. First of all, since state variable risk is not priced in a constant relative risk aversion framework, a necessary ingredient in these models is the – albeit small – risk of disastrous consumption shocks. In contrast, empirical time series of consumption appear to be rather smooth and do not exhibit such enormous shocks. A model
with recursive utility and priced state variables might therefore be more promising to match empirical data.

Moreover, from one-tree models, it is well known that, in an economy with a representative CRRA investor with relative risk aversion above 1, asset prices are a decreasing function of the expected dividend growth rate and an increasing function of the dividend volatility which is contrary to the natural intuition. Indeed, the paper of Branger, Kraft, and Meinerding (2011b) exhibits this natural failure, and all asset prices increase upon a crisis. The paper of Buraschi, Porchia, and Trojani (2010b) circumvents this problem artificially by assuming that expected dividend growth is higher if a firm is in distress so that asset prices indeed decrease after a distress.\(^8\) In a model with recursive utility, this shortcoming could be resolved.

5 A brief outlook on future research opportunities

As the present literature overview shows, a considerable effort has been made during the past few years to shed light on the link between systemic risk on the one hand and asset allocation and asset pricing on the other hand. Nevertheless, there is still a long way to go. Perhaps the most interesting starting ground for future research is provided by the model of Ait-Sahalia, Cacho-Diaz, and Laeven (2010) which is itself tightly linked to current models of portfolio credit risk.

Concerning the asset allocation problem, a combination with the filtering results of Branger, Kraft, and Meinerding (2011a) should be able to answer several open questions as pointed out in Section 3. First of all, the effect on an investor’s portfolio choice is not clear at all since Ait-Sahalia, Cacho-Diaz, and Laeven (2010) cover the theoretical solvability only. It would thus be interesting to see a calibrated version of the model quantifying the impact of contagion on optimal portfolios.

Secondly, it is by no means clear how one should introduce uncertainty into the models. Filtering unobserved drift rates out of the observation of diffusive stock prices is

\(^8\)Loosely speaking, a minus times a minus equals a plus, i.e. switching the sign twice, we are back in the proper direction.
computationally tractable. However, filtering Poisson jump intensities can well become hard as the work of Frey and Runggaldier (2010) shows exemplarily. If the jump intensities are driven by a Markov process with continuous state space or if one deals with Hawkes processes, filtering will be even harder. To circumvent these difficulties, it might be promising to have a closer look at the work of Branger, Kraft, and Meinerding (2011a). They point out that, in terms of expected utility, filtering from jumps only is sufficient for a CRRA investor. In this sense, it might be possible to solve a model with a simpler, 'suboptimal' filter and to compute the optimal portfolios and expected utilities therein.

Moreover, the impact of systemic crises on life-cycle portfolio planning problems seems to be a field which has not been investigated at all. In such a setup, the hedging demand against contagion risk influencing the investment opportunity set might serve as an explanation for asset allocation puzzles like the underdiversification puzzle (in the sense that diversification does not prevent losses from systemic events anyway) or the stock market participation puzzle (in the sense that the fear of financial crises reduces the stock market participation rate significantly).

The asset pricing results presented above mainly suffer from the assumption of constant relative risk aversion utility which makes it impossible to match any empirical figures in Lucas orchards. The major shortcoming originating from this assumption is the fact that state variables affecting consumption do not carry a risk premium. Therefore, all variation in asset prices must arise from variation in the consumption itself. Since empirical time series of consumption are rather smooth, it would be more appropriate to model contagion via disasters in underlying state variables instead of disasters in consumption and dividends.

A major advancement could thus be made if recursive utility was embedded in an asset pricing model with systemic risk. This would also help answer the most intriguing question: What is the market price of contagion risk? Since state variables are not priced in a framework with constant relative risk aversion and no paper with recursive utility, Lucas orchards and an appropriate notion of systemic risk is currently at hand, this question has not been answered to date.
Furthermore, a correct model-theoretic notion of contagion is indispensable. Presumably, the literature on portfolio credit risk, unobservable default rates and self-exciting jump processes can help here. One could, e.g., extend the economies of Branger, Kraft, and Meinerding (2011b) by more sophisticated jump dynamics, more economic states, more than two assets, uncertainty about jump distributions, stochastic jump sizes or a more realistic framework for default clustering. To this end, a combination with the ideas of Ait-Sahalia, Cacho-Diaz, and Laeven (2010) and Buraschi, Porchia, and Trojani (2010b) seems appropriate.

Once the link between time series of stock prices and systemic risk is worked out properly, the next issue would be to investigate cross-sectional price impacts of financial contagion. And last, but not least, an extension towards other asset classes like bonds, plain-vanilla options, variance contracts, or pension funds would be a challenging task for future research.

References


